Yes, we can!
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Everything but ordinary!

Authors:
Bernadette Wieser
and Anita Hotter,
Down Syndrom Centre
Live Laugh Learn,
Leoben, Austria
Everything extraordinary? Yes! Everything but ordinary!

It is tremendous, esteemed reader, that you have taken this book in hand to study. For in so doing you have taken the first step in supporting individuals born with Down Syndrome as they journey towards developing an understanding of so-called everyday mathematics. Yet the most important step still lies ahead: You must in fact BELIEVE that they will arrive at their destination. YOUR FAITH in the ability of individuals with that certain Extra, also referred to as Down Syndrome, is the most crucial element for success! Over the years, we, the authors of this book, have had the privilege of accompanying hundreds of individuals with Down Syndrome, from the most varied of age groups, as they have progressed mathematically. Hundreds of unique personalities pursuing their own individual paths of instruction. In the following we will speak of “pupil and teacher”. A couple of brief explanations: By “teacher” we mean, of course, all of those who live and work with individuals with Down Syndrome, whether parent, therapist, kindergarten trainer, early interventionist, assistant etc. “Pupil”, in turn, refers as a whole to anyone with Down Syndrome, be it child, adolescent or adult. In order to balance fairly both feminine and masculine pupils and teachers, the choice of gender in this book varies from chapter to chapter; similarly, the photographic material was selected purely coincidentally. Regardless of whether boy or girl, man or woman: Mathematical development in no case begins with the first attempts to count or to calculate. The roots lie in early childhood, in the development of a well-defined body language. This book provides numerous suggestions for games that support fullgrown development of the so-called basic skills. These include first and foremost body language, visual and auditory perception, spatial orientation and seriality. By all means please begin training in these areas first, for without a foundation, no house can be built. All suggested exercises adhere to the principle “from easy to difficult”.

What else is important? Before beginning work with your pupil, you must first determine the stage of development of his basic skills, as treated in the first chapters of this book. You and your pupil should together become acquainted with the materials of the “Yes, we can!” kit. These are clearly organized and simply fashioned with respect to colour, form and aesthetic, and their use is suited to the most diverse levels of learning. It would be best to try out the individual game suggestions. Those games that would appear to be rather tricky for your pupil would in fact be most suited to challenge him. Work through the chapters one at a time and match the degree of difficulty to the degree of development (not to the age!) of your pupil. Older pupils in many cases also require oppor-
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as the fingers and hands, a sophisticated language of the hands can be developed, and together with that, those areas of the brain that are responsible for calculating. The finest result of this is the fact that the calculating materials are always immediately present, as our hands accompany us step by step. In each and every daily situation they are there for us and... they can never be lost! Forever and everywhere, exactly 10, divided by two times 5, a more practical solution could not exist.

Most individuals with that certain Extra benefit from visual learning opportunities. By observing and imitating, they achieve “understanding”; movement supports the development of their grasp of the physical concept of “grasping”. By employing physical components such as the fingers and hands, a sophisticated language of the hands can be developed, and together with that, those areas of the brain that are responsible for calculating. The finest result of this is the fact that the calculating materials are always immediately present, as our hands accompany us step by step. In each and every daily situation they are there for us and... they can never be lost! Forever and everywhere, exactly 10, divided by two times 5, a more practical solution could not exist.

In those games that help to develop basic skills, exchange roles as often as possible: The pupil becomes the teacher, you become the pupil. This offers both parties exciting perspectives, expands the behavioural repertory of your pupil and teaches him responsibility. Together with furthering the development of basic skills, school-age children, adolescents or adults can also be introduced to the first exercises in counting with their fingers, for which you will find numerous incentives in the chapter “Counting to ten”. Finger games support the development of coordination and refined motor skills. It is, however, vital that you first thoroughly acquaint yourself with the methodology involved. The video “Yes, we can!” offers step-by-step instructions for the technique of counting and calculating with the fingers.

Please bear in mind that the mental concept of “grasping” was developed from the physical concept of “grasping”. By employing physical components such as the fingers and hands, a sophisticated language of the hands can be developed, and together with that, those areas of the brain that are responsible for calculating. The finest result of this is the fact that the calculating materials are always immediately present, as our hands accompany us step by step. In each and every daily situation they are there for us and... they can never be lost! Forever and everywhere, exactly 10, divided by two times 5, a more practical solution could not exist.

Most individuals with that certain Extra benefit from visual learning opportunities. By observing and imitating, they achieve “understanding”; movement supports the development of their grasp...
of basic mathematics. The approaches to and procedures within the realm of numbers are multiple. As parallel to the introductions to the Feuerstein, Montessori and Numicon concepts, this book treats the experimentally-proven method of “Yes, we can!” as but ONE path towards the objective. This is highly efficient! Try it out and together with your pupil enjoy progress and success.

Only a relaxed brain can learn! For this you need a familiar setting, a great deal of laughter and a warm personal contact to your pupil! Only on the basis of mutual trust is it possible to progress and devote oneself to new material. Inspiration grows out of the individual approach to the subject. Always take into consideration the interests and hobbies of your pupil, for it is here that one identifies! What does he like to collect, what figures of music, sport, film, comedy does he idolize, what transports him into a flow-experience? Once you discover these connections, you need not worry yourself anymore about how to motivate your pupil to calculate. The positive associative feelings spill over automatically into mathematics.

What is decisive is that the instructional material be combined with that which is already familiar to the pupil from situations in his own life. The phrase “That is, for instance, like…” can open the door to the realm of numbers. With respect to mathematical content there are many points of reference to be found in the everyday, familiar world of the individual with Down Syndrome, from shopping to reading the time in a TV program, from the use of a cell or mobile phone to the correct apportionment of strawberries among siblings. His own pocket money for the cinema, or self-earned funds for an excursion: “Everything is number”, as Pythagoras already knew. Instructional content that nevertheless cannot be linked either to pre-existing knowledge or to actual situations in life, to the interests and hobbies of the pupil with Down Syndrome, flow through his brain as through a sieve. Instructional offerings that are rejected by the individual with Down Syndrome do not link up to his world. If you wish, you might create a “calculating driver’s license” for your pupil. This “official document” should include his name and photo, and preserve a record of all that he has achieved. The path is the objective.

Dear reader! You know that even the longest journey begins with the first step. You ventured forth in the moment you took this book in hand. Please do not stray from the path!
Evolvement of body language

Jana, a six-year-old girl with Down Syndrome, describes herself as an octopus with large eyes. She is presently learning to distinguish and name her own body parts and those of others. Since she is constantly running into the edges of tables and tripping over her own feet, she particularly benefits from exercises that make her aware of her own physical dimensions.

Mosquito bite

Name the parts of the body and apply a lotion to them, massage them with a soft brush or tickle them with a feather. With this activity, those body parts that are present in pairs, such as the ears, eyes, arms and legs, are taken up one after the other. Place cold or warm packs (from the pharmacy) on individual parts of the body. Mosquito bite: Jana’s playmate takes her finger and lightly pricks Jana on a part of her body. Jana is supposed to find and name this spot on her own body and on her playmate’s body.

Eyes closed

The ability to close our eyes briefly when wide awake requires a great deal of trust in our senses. In particular, the sense of sight gives individuals with Down Syndrome a feeling of security. Being able to shut down this sense for short periods fosters the development of internal imagery and in turn the capacity to think abstractly. As training, a large box filled with pillows is a great help to the pupil. She settles into her “nest” and is surrounded by barriers on all sides. Now she closes her eyes for a few seconds; perhaps a strain or two of music is played in the background. Once she feels really secure and can keep her eyes closed for some 10 seconds (depending on the circumstances, this can take weeks or months), one can cut away that part of the box at her feet, then later, one after the other, both sides, and finally the back. Now the pupil lies on her pillows and can keep her eyes closed for almost a minute. Perhaps during this period she listens to a story or a piece of music?
Singing, dancing, clapping, casting shadows

Many songs, clapping games, dances and physical exercises make us conscious of the individual parts of our own bodies in an amusing fashion, and also teach us about energy reserves and the planning of motions. For all of these activities we must take particular care to include both hands. With respect to rudimentary motions, climbing, hopping and a game of ball require the coordinated interplay of our senses, our brain and our muscular system. When casting shadows, the pupil stands next to her teacher and imitates her movements. All movements should initially be limited to one side of the body (e.g. extending from the right elbow to the right thigh), but later they can cross the midline (e.g. left thumb to right ear).

Quick fingers

Eye-hand coordination is sharpened by all refined motor activities.

A few ideas for training:
- paint individual fingers with finger paints
- unravel woollen balls
- shred newspapers, crumple them into a wad and toss them (successfully) into the wastebasket
- hang up the laundry with clothes-pegs
- play fishing games
- play string games
- make a chain using paper clips
- touch fingers (slow and fast): thumb to index finger, thumb to middle finger, thumb to ring finger, thumb to little finger
- shoot marbles or small stones through a goal, using various fingers
- finger theatre: all are asleep (make a fist), one after the other awakes (extend the fingers one after the other) and rouses the next (the extended fingers tap those that are still tucked in)

- "Paper, scissors, stone":
  "Paper" is represented by five extended fingers; "stone", by a fist; "scissors", by a V formed by the index and middle finger. After the signal "go", each of the two players displays a hand to represent one of the three symbols: either that for paper, for stone or for scissors. Paper beats stone (because it can wrap up the stone) Stone beats scissors (because it can pulverize the scissors) Scissors beat paper (because they can cut the paper to pieces)
  Who has won the most points after 10 rounds?

- cut, fold origami, knead
- crochet, knit, tie bows (see also "Understanding seriality"), wrap packages
**Treasure hunt**

The pupil fills her bag with everyday objects. She then tries to find particular ones by feeling for them.

**Drawing and writing**

The development of hand language is particularly important with respect to writing letters and numbers. If the pupil still grips the pencil using the three-point system (thumb, index finger and middle finger), or in fact simply with her fist, this can lead not only to muscle cramps but also to the loss of a desire to write.

To write or to draw in a relaxed fashion, the pencil should be held with the thumb and index finger bent in the form of pincers. Either a pencil holder or shorter pencils will gently guide the fingers into the correct position.

The pincer grip grows out of the tweezers grip. How does one practice the tweezers grip? By picking up smaller objects, possibly initially assisted by the teacher.

Practicing, and then eating, is fun: Short pieces of pretzel sticks, small berries or sunflower seeds can also perk up the taste buds.

**Feeling and calculating**

Eyes closed! Which hand is being touched? Which finger is being touched? The ability to divide our two hands into left and right and to consciously sense and name the individual fingers assists the development of our calculating skills. In this case, it is not important that the fingers be identified as “thumb, index finger, middle finger, ring finger and little finger” or by number. What is important, however, is that the eyes remain closed during the game (or that the fingers remain hidden under the secret box, and that the pressure placed upon them is not too soft, but nevertheless gentle.

The two hands are outlined on a sheet of paper. The teacher then points to one finger after the other, which the pupil must find again on the sheet as well as on her own two hands.

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10  **Yes we can!**
I’m the boss!

At the moment of birth the differing proficiency of our two hands is already determined, which leads to domination of the one hand that will bear the burden of work—it is assisted by the auxiliary hand. Around 30 to 40 percent of mankind is born left-handed. Yet in many cultures substantially fewer persons write with their left hand. Why is this so? The main reason may well be that children exactly imitate those around them. If they are reared in a household where right-handedness dominates, the likelihood is great that they, too, will tend to be right-handed, even if they were born left-handed. No child should be consciously retrained! The hand we favour is innate! Retraining of any kind can cause irreparable damage during childhood development. By the time a child enters school, one hand should have assumed dominance, and it is this hand that takes the lead in complex coordinating feats that demand refined muscular skills (e.g., writing).

Which spontaneous, untrained actions provide tell-tale clues that help us to determine which hand naturally dominates?

- opening and closing doors
- winding up toys
- watering flowers
- spinning tops and rolling dice
- toppling stacked tins with a ball
- shooting marbles
- hanging up the laundry with clothes-peg
- shredding and crumpling paper

For activities such as eating, brushing the teeth, cutting or painting, one should take care that the necessary utensils are centred directly in front of the child.

Jana’s parents have made an arrangement with her kindergarten teacher and her grandparents that they will observe the child’s use of her hands for a period of one month, and that they will keep a written record of their observations. If at the end of this period there are definite clues as to which hand dominates, this will be made clearly visible to Jana by means of a bracelet. When performing her writing or cutting exercises or when eating, Jana will be alerted to use the hand that is “the boss”.

body language
Behind, in front of, above, below, beside:

everything round about me!
**Understanding spatial relationships**

Jan has already looked behind the plant, between the chairs, under the table and on the sofa. There, he finally finds his brother Jens: He is in the closet. The two boys both have 93 chromosomes – and all kinds of fun playing hide-and-seek. Totally incidentally to this they are developing fundamental mathematical understanding, which is, of course, based on orientation within the number system. An important prerequisite for this, however, is two- and three-dimensional spatial orientation. The following exercises should help to improve and develop this further. These exercises align themselves to the principle of progressing “from the simple to the difficult”. They therefore gradually proceed from three- to two-dimensional perception, from the concrete to the abstract.

**Hide-and-seek**

Using his own body as point of departure, the pupil is to discover his immediate relationship to objects: Hide himself, hide a stuffed animal, look for someone. All positions within a particular space should be described aloud – either by the pupil himself or by the teacher on his behalf. “I am hidden under the table. You are hidden behind the door.” Vocabulary and spatial perception are amplified by the use of positional descriptions. In group games the pupil should always consciously climb over (or crawl under) obstacles, all the while speaking aloud! “Through, in between, underneath, on top of, above, below, right, left” etc. will be discovered in the course of the activity. Important is the clear emphasis on the description, accompanied by simultaneous action.

**From here to there**

The teacher instructs the pupil to proceed to a particular position. “Take three steps forward, now turn towards the window, take one step in the direction of the door…” etc. A little surprise awaits him at his destination. Later include the concepts of left and right. To simplify this perhaps mark the dominant hand of the pupil with a bracelet.

**Table, set thyself**

Setting a table for several persons requires discriminating spatial orientation. The pupil sets the table and accompanies the steps speaking aloud: “The plate is positioned in the middle. To the left is the fork” etc.
Build a tower

The pupil erects buildings according to pre-existing models using stones, Lego blocks or clay.

Father, mother, all present!

Children enjoy this game, whereas the next game (Ball, triangle, all present!) is more suited to adolescents and adults. The teacher shows the pupil a photo that contains various positional designations, such as: “in front of – behind – on top of – below – next to – inside of – in between”. The pupil duplicates this photo using actual figures.

The pupil begins with two figures that are placed next to each other. Then he copies additional photos with 3-6 figures that are placed together as if representing a particular situation. As a rule, the placement of the figures initially adheres to the original model. Later the figures should be placed ca. 10 cm apart. You will find the photos illustrated in the corresponding sequence, that is, from easy to difficult.

Ball, triangle, all present!

This game is somewhat more advanced than “Father, mother, all present!” in that it uses geometrical forms instead of figures. The instructions are the same as above. Begin with two forms that are placed next to each other.

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Furnish a house

The calculating kit includes loose materials that can otherwise be used for furnishing a house of 6 to 9 rooms (depending on the desired stage of difficulty). An everyday object is provided for each room. Following the teacher’s instructions, the pupil now furnishes one room after the other. An example: “Place the key at the top left. Place the peg at the bottom in the middle.” If the pupil has difficulties, it is recommended that the teacher build a second, pasteboard box of very similar size, and that he fill this with objects as the pupil looks on. Thus: The teacher places the pocket handkerchiefs at the top right and says: “I am placing my pocket handkerchiefs at the top right.” He then gives the pupil a second package of pocket handkerchiefs and says: “Please place yours at the top right, too.” It is more difficult if 9 rooms are to be furnished. As the pupil furnishes his house, he describes the process aloud, if possible. If he does not have the ability to express himself well enough, the teacher will do this for him.

At the next level of difficulty, the teacher photographs the fully-furnished house and the pupil fills all of the rooms to match the photo. It becomes even more difficult if the pupil has to furnish the house using only a plan.

Follow the example

A line is drawn vertically, as if in a standing position, on a piece of poster board that is then pasted to the door (in such a way that the pupil can stand next to it). A line of another colour is drawn horizontally, as if in a reclining position, on a second piece of poster board that is then placed on the floor. Upon hearing the signal “standing” or “lying”, the pupil proceeds to the corresponding poster and either stands next to it or lies down. It is more difficult if the instruction is not spoken aloud, but indicated by the teacher with her index finger positioned either vertically or horizontally, or by a small card displaying either a horizontal or a vertical line.

Magic wand

On a work sheet, vertical and horizontal lines have already been drawn with blue, removable ink. Following directions (“horizontal line, vertical line”, or as a descriptive aid, “lying line” and “standing line”), the pupil removes the line with the magic wand, which is actually an ink eradicator.

Reading backs

A horizontal or vertical line is drawn on the back of the pupil, who traces this line either on the table or in the air.
roomorientation

Copy the pattern

Using materials from the calculating kit, the teacher lays out a pattern with 3 tens-sticks (or tens-rods). The pupil lays out the same pattern next to this. Depending on the pupil’s progress, the number of sticks is increased. As preparation for the writing of numerals, it is very important that the pupil observe exactly whether the sticks are placed horizontally, vertically or at an angle. This exercise can also be practiced with toothpicks. If there is uncertainty, it is helpful to paint the tips of the toothpicks for better orientation. Expand the exercise to include other types of materials (drinking straws, Mikado sticks etc.). The worksheet in the Appendix combines two- and three dimensional tasks. Using the tens-sticks, the pupil imitates the patterns displayed. Should he encounter difficulties, he may place the sticks directly on the pattern. Pros imitate the patterns from memory!

Flight to the sun

With this worksheet, the pupil should try to draw the correct path for the airplane to reach the sun. Initially, it is important to talk over the exact flight path with him and to assure that every single leg of the journey is described. For instance: “The airplane first flies to the right, then downwards for a while, then continues left” etc. This aids the pupil’s orientation. Gradually, however, he should try to work out the path alone.

Once round about

4 objects are placed on the floor like corners of a trapezoid. The pupil stands in the middle and names the objects, with their respective positions, one after the other, e.g.: “The mobile phone is in front of me. The key is to my right. The cup is behind me. The book is to my left.” The pupil then turns 90 degrees clockwise and names the “new” position. “The key is in front of me, the cup is to my right” etc. Afterwards the pupil makes two more turns of 90 degrees each, and finally returns to the original position.
Carousel

On a sheet of paper the teacher draws a chart with 3 rows of 2 columns each. Using the forms from the form sack, she places objects of the same colour on 5 of the squares and a form of another colour on the 6th square. This position is now identified by the pupil, e.g. “top left”. Now the sheet is rotated clockwise 90 degrees. The position has changed and must be newly identified.

Level for level

The rectangular board from the calculating kit contains 9 pegs. All of the boards have one loose peg (its position is marked on the back of the board) that the pupil can remove and then reinsert. This position is discussed exactly and then the board is turned a quarter revolution; i.e. the loose peg has changed positions – e.g. from the original position of “bottom middle” to “middle right”. This rotation and the relocation of the loose peg particularly challenge the imagination. The difficulty increases when a second board is placed atop the first. Now the pupil must note the position of the loose peg on every level.
Auditory perception

“Did the teacher just say forty or fourteen?” Marc cocks his head to the side and knits his brow. To be able to distinguish words that sound similar demands the full concentration of the 12 year old. His trademarks: Freckles, football fan, 47 chromosomes – and an earring, which unfortunately does not help him to understand instructions and explanations any better or to distinguish sounds more readily. As many of his cohorts with that unusual constellation of chromosomes known as Down Syndrome, Marc requires well-directed assistance as he strives to acquire the skill of phonemic differentiation, the ability to perceive and process acoustical stimuli.

Auditory memory represents a special challenge for individuals with Down Syndrome. They often poorly understand incidental remarks, particularly against a background of distracting noise.

How can one help?

• When the teacher speaks with the pupil, he should do so at eye level and should look the pupil in the eye.
• The teacher requests the pupil to repeat the most essential parts of what has been said.
• The teacher accompanies his statements with hand gestures. These can be agreed upon between him and the pupil. In this way, the pupil not only hears the content, but also sees it.
Noise everywhere

Positioned behind the pupil, the teacher creates various noises. The pupil is to identify these and imitate them.

- crumple various types of paper
- cut, tear paper
- knock on, rub, scratch various types of objects (wood, metal, synthetic materials)
- let water drip
- produce noises of motion, such as hopping, crawling, jumping, walking
- produce various noises with your mouth
- produce various noises with your fingers

It is also amusing to toss various unbreakable objects onto the floor (e.g. a pencil, a rubber eraser, a stuffed animal, an exercise book, a box…). The pupil has to guess what he has heard.

Noise maker

Various objects that create noise are hidden in a room (an alarm clock, a mobile or cell phone, a stopwatch, among other items). The pupil tries to find the noise maker. The task is more difficult if there is soft music in the background.

That’s my name!

The teacher tells a story that frequently incorporates the name of the pupil. When the pupil hears his name, he raps with his hand. Adolescent and adult pupils are read a text that interests them. At the sound of a word agreed upon previously as a signal, they rap on the table.

Sound quiz

Each of two cups in a series of cups is filled with identical material (uncooked rice, dried peas, nails etc.). It is the pupil’s task, after shaking the individual cups, to determine which two cups belong together.
Mickey … Mouse

The rule is simple: The pupil stands before a string stretched out on the floor. “Mickey” lives in front of the string, “Mouse” lives behind the string. If the teacher says the word “Mickey”, the pupil jumps into Mickey's house. If the teacher says “Mouse”, the pupil jumps into Mouse's house. So far so good. Now the teacher calls out the two words in random order. Beware: If the pupil is already in Mickey's house and the teacher calls out the word “Mickey”, the pupil must remain where he is; he is not allowed to jump. This amusing game trains reflexes and the ability to concentrate on acoustical stimuli, accompanied by an enormous amount of fun! Adults might very well prefer to use a couple of names from a music group.

Listen exactly and count along!

The pupil either closes his eyes or is blindfolded. The teacher drops pearls into a cup, the pupil counts along and identifies the number simply by listening.

Knock, knock

The pupil is to associate various types of knocks with specific physical movements, e.g.: one knock means “nod”, two knocks means “clasp the hands”, three knocks means “make a fist” etc.
Secret telephone number

The teacher slowly states two numbers one after the other. The pupil repeats these “secret telephone numbers”. If the pupil masters this task independently, increase the numbers to three or more. It is particularly tricky if the numbers happen to be stated in reverse order. This is then truly a secret number!

I beg your pardon?

On two separate note cards the teacher writes down two numbers the pupil already knows, e.g. 4 and 8. On a third card he writes down a number the pupil does not yet know, e.g. 19. He shows the pupil the three cards and says “8”. The pupil points to “8” on the card. Then the teacher states the second familiar number, namely “4”, and the pupil points to “4” on the card. Finally the teacher calls out the unknown number, “19”, to the pupil. How will the pupil respond? If he points to “19”, he has demonstrated, through the process of elimination, significant competence with respect to logical inference. In this fashion the number vocabulary of the pupil can be steadily increased.
The recognition and differentiation of stimuli that are perceived visually is often an outstanding skill of individuals with Down Syndrome. David, who has that extra chromosome, was already capable of holistic word recognition and identification even in his pre-school years. Today, at the age of eight, he is a reading pro. Visual perception lays the foundation for the greater part of our actions and also plays a decisive role in the development of our calculating skills. To be able to filter out what is important from a confused backdrop is achieved by so-called figure-field perception. David opens up his math book and must now concentrate on one specific problem. In the meantime, all other numbers, functional signs, explanations and diagrams must be filtered out of his brain. The fact that David can grasp that not only a small quadratic plate, but also a book, a window or a CD cover are rectangles owes to the constancy of his perception. This is an important prerequisite for our ability to recognize numbers and functional signs when reproduced in various fonts and sizes. And David’s ability to store the shapes of numbers for a long period of time is aided by his visual memory.

Comparing and categorizing
To categorize means to combine things in a group. When we tidy up, we apply this strategy. Likewise when we learn which letters are to be written as capitals and which numbers belong within the range of 10. Countless opportunities to categorize in daily situations prepare the pupil for this essential requirement of daily existence. You as teacher should therefore also mention again and again the primary categories. “The car, the bicycle and the truck are types of vehicles”. Categorizing goes hand in hand with comparing. It is the ability to be aware of perceptions in the most diverse of situations, in order to deduce generally valid rules (e.g. “apples are round, bananas are longish”). Here the comparison of objects represents the first step towards inductive reasoning and thereby assists us to arrive at practical solutions, to plan logically and to develop unproblematic strategies. The capacity to compare, to differentiate and to categorize lays the foundation for the understanding of equations. The pupil should become acquainted with categories that are suited to visual comparisons, e.g. form, colour, size etc.
Look for pairs

Select 2 identical buttons, socks, ballpoint pens, bowls, spoons etc. from an array of many.

Order, please!

Organize by groups items in the toy chest, in your own clothes closet, on the desk or in the CD rummage box:
Countless opportunities to categorize in daily situations prepare the pupil for this essential requirement of his academic day.

Experience forms

In order for the pupil to learn various forms, these must first be comprehended at the two- and three-dimensional level. A circle or a line is drawn on the pupil’s back, which he is to reproduce on a sheet of paper in front of him. When introducing the rectangle, it is essential to begin with large rectangles on the floor or on the wall. Masking tape is used to lay out a rectangle on the floor/on the wall and this is consciously traced with the fingers, while speaking aloud: “down – stop – across – stop – up – stop – close – stop”. The word “stop” is crucial since it assists the pupil to make the transition from the form of the circle to the form of the rectangle. Equally well on the floor the pupil can hop or crawl along the edges of the rectangle – while speaking aloud. Whether sand boxes, shaving cream, snow, forms laid out with sticks or branches: All are suited. Pay particular attention to forms that are encountered in everyday situations, and have the pupil touch these, e.g. the top of a glass is round – the pupil runs his finger along the mouth of the glass. Or a book is the shape of a rectangle – the pupil touches the edges of the book etc.

V4. Sort forms at home

Sort plates, cups, clocks (‘round like a plate’), books, papers, boxes, the telephone (‘rectangular like a book’) etc.

Mother and child

The teacher selects a form from the bag ("mother") and places this on the table. The pupil selects the same, smaller form ("child") and places the forms side by side. If, in the process, the pupil uses spaghetti tongs or sugar tongs, finger dexterity will likewise be trained. Should a pupil have trouble finding the same forms, the number of items from which the form is to be selected should be reduced. Initially the pupil might select from only three forms, later from 10, and at the level of pro from forms of every type.
Yes we can!

**Visual perception**

**Match forms**

The pupil should, for instance, be instructed to sort a form in 3 colours, then another form in 3 sizes. On a sheet of paper the teacher traces the outlines of various forms that are to be found in the bag of forms, and the pupil should then select the forms that match these.

**Large and small**

Using 2-6 small forms from the bag of forms, the teacher designs a figure on the table. The pupil selects the same forms in another size and copies the original figure.

**Play bingo with forms**

Using his own sheet of white paper, each player outlines six forms from the bag of forms: In this case he may choose any form the bag contains (triangle, rectangle, circle of any size). After the players have finished preparing their bingo cards, all of the forms are returned to the bag of forms. The teacher now draws a form. Whoever finds this form on his sheet of paper may paint it. Who is the first player to have painted the entire sheet?

**Feel forms**

For this activity use a small empty sack from the box and fill it with a variety of forms. The pupil closes his eyes, reaches in and takes a form; as he does so, he should feel it and make a guess as to which form it is.

**Angular shapes, go to the door**

Each player takes one form in his hand and observes its exact shape (round, rectangular, triangular). The teacher gives directions, e.g.: All angular shapes go to the door, all circles sit down on the floor, all rectangles fold their hands.

**Traffic signs**

Private eyes, up front! Equipped with a camera, pupil and teacher go in search of forms that are hidden in traffic signs. Where do we find a triangle, a rectangle or a circle in traffic? Pupils who are still seriously distracted by vehicles in traffic should use pictures of traffic signs to look for the forms.
I see something you don’t see

The teacher presents the pupil with a little puzzle: “I see a small, round, yellow form”. The pupil tries to identify and name the correct form among various others lying about on the table. “I see a small, green form with four corners”... If the pupil is verbally capable, he should also present the teacher with a quiz question. This is a great challenge to both players.

Point for point

On the work sheet the pupil connects the dots to create a geometrical form. As soon as he recognizes it, he names it (if possible) and selects the matching piece.

“equals”

The symbol “=” (equals) is introduced to the pupil in an active, concrete situation. The teacher prepares the following card:

The pupil lays two small, blue ones-cubes (or two red tens-sticks) on the table and between them the card: Afterwards the pupil selects identical everyday objects and equates these with the card.

It is very important that the teacher discuss with the pupil why the objects are identical. What criteria were used for comparison: Colour, form, size, material, function?

“does not equal”

If the pupil finds identical forms and groups these together, it is just as important that he find non-identical forms. The teacher again discusses with the pupil the reason that the forms of the circle and the rectangle are not identical (whereas the colour or the size might agree).

For making comparisons one can use numerous objects that the teacher has prepared as well as things that the pupil finds about him. When we instruct pupils, we frequently assume that the terms we use are understood in the sense that we mean them. Particularly terms that express comparisons are often not well embedded in the memories of individuals with Down Syndrome. Offer your pupil a great many opportunities from day to day to make comparisons and to learn the characteristic expressions for what is long, short, fat, thin, high, low, wide, narrow etc. By example of numerous comparisons we can create a sound basis for the understanding of relationships, such as “more/less”, and their intensifications, such as “long/ longer/ the longest”. And not only visual characteristics can be analyzed, but also, for instance, volume of sound, surface textures, temperature, taste and odour. Likewise popular find-the-error picture-games presuppose the ability to make comparisons!
Understanding seriality

Eight-year-old Marc, with that certain Extra, has to concentrate intensely on how to tie a bow with his shoelaces. It is not that he is having trouble with his coordination, as this is sophisticatedly developed. It is the order of events: What is the first thing he must do, and what comes afterwards? A clear repetition of the instructions will help him overcome these difficulties. In order to absorb and digest the overwhelming forces of the world about us, we must possess the ability to assort them according to time and hierarchy. The ability to organize sequentially and to recognize regularity is referred to as a serial achievement. We are obliged to observe sequences not only in everyday situations, but also when reading or writing things down. Seriality likewise plays a decisive role in mathematics, e.g. when counting, for understanding the number array (or number line), and also when calculating. To lay the foundation for understanding seriality, one must begin at the concrete level, in order to progress from there to the pictorial level and ultimately to the abstract level.
Yes we can!

Clap, stomp, wave, catch

The playmates toss a soft ball or the small pillow from the calculating kit to each other. But careful: Clap before catching! If this works well, add another movement: Clap, stomp, catch. After several successful rounds, further movements may be added, for instance, wave, turn, tap a foot etc. Can the pupil recall the correct sequence? It is important to increase the series of steps gradually!

From head to toe

The teacher points to 2 parts of the body one after the other: Eye- stomach. The pupil now touches the respective parts of his own body in the correct sequence.
Gradually increase the number of body parts to 3.
If the body parts are only spoken aloud and not pointed to, the degree of difficulty increases.

Lay out a pattern

The pupil collects objects in the woods, such as stones, twigs, leaves, pine cones, with several similar pieces in each group. The teacher lays out a pattern and the pupil should continue the sequence.

Picture frames

A picture of the pupil is glued to the middle of a sheet of paper. A frame is drawn around the picture. The teacher uses stickers to lay out a pattern on the frame. The pupil takes over and continues the pattern.
Blue, green, red

The teacher removes the coloured circles from the sack of forms and places them on the table. He then calls out a sequence of colours, e.g. “red – blue”. The pupil responds by trying to hit the circles in the correct sequence with a fly-swatter. Once he is quite sure of himself, the number of circles should be increased to 3 (“yellow-blue-red”) and later to 4 (“red-blue-red-yellow”).

Photo stories

The pupil is photographed carrying out everyday activities, e.g. brushing his teeth:

- Photo 1: Toothpaste and toothbrush
- Photo 2: Brushing his teeth
- Photo 3: Rinsing
- Photo 4: Putting things away

The pupil arranges these photos in the correct sequence. Little by little the number of photos should increase.

How was this picture produced?

Using several small cards, the teacher draws a house.

- on the first card is the floor and one wall,
- on the second card is the floor, two walls and the roof,
- on the third card is the floor, two walls, the roof and the door,
- on the fourth card is the complete house.

The “Story of the Construction of the House” is presented to the pupil in the wrong order and he arranges the cards in the correct sequence.
Sing with me

To help develop acoustical seriality, the teacher sings a certain number of syllables for the pupil, e.g. “la – le – li”. The pupil tries to repeat these in the correct sequence. If this is too difficult, reduce the number of syllables to 2; if the pupil is very secure, increase the number accordingly.

Clap with me

The teacher claps out a rhythm for the pupil, e.g. long-long-short. The pupil tries to repeat it in the correct sequence. If this is too difficult, reduce the number; if the pupil is very secure, increase the number accordingly. Game variations: Musical instruments are exceptionally well suited for this game, but there is nothing to stand in the way of one’s own improvisations (e.g. pot and spoon etc.). […]

From short to long

The pupil arranges the wooden sticks from the calculating kit in the order of shortest to longest. Coloured pencils or straws cut to various lengths can also be arranged in sequence according to their length or thickness.
1, 2, 3, 6, 4! Anna, 5 years old, eagerly counted the steps in front of the entrance. When children begin to count, they simply recite the row of numbers – regardless of quantity – at first in no particular order, later in correct sequence. Their counting skills thus initially develop independently of aspects of quantity. At this stage enumerating is still not possible. Anna’s next step in the learning process is to form an “unbreakable chain” from this undifferentiated “number potpourri”. In the process she must learn to recognize that quantities are hidden within the numbers. If enumeration is performed correctly, a link is created between the number and the object: Counting competence awakens.

Parallel to counting, many pupils develop a great interest in numbers and their designation. In everyday situations there are constantly opportunities to discover numbers and to identify them by name.
Counting with the fingers: from left to right

All children begin to count using their 10 fingers. As part of their own body, these represent the most original visual aid and link the tactile-kinaesthetic and visual worlds of senses. Counting with the fingers creates a link from numbers and quantities to the realm of the body, to the realm of objects and to the realm of numbers.

The number array is a linear organization of numerals. Beginning with 0, the whole numbers are arranged from left to right according to size. The arrow signals the open end at the right.

Exactly as arranged in the number array, teacher and pupil together start off on their joint counting venture, based on the following principle:

- The left little finger represents 1.
- The left ring finger represents 2.
- The left middle finger represents 3.
- The left index finger represents 4.
- The left thumb represents 5.
- The right thumb represents 6.
- The right index finger represents 7.
- The right middle finger represents 8.
- The right ring finger represents 9.
- The right little finger represents 10.
Advantages

This manner of counting (and later of calculating) supports the development of our spatial orientation from left to right, which our culture also expects of us when we read and write. Furthermore, it lays the foundation for the development of a mental number array, whereby we mean our spatial and automated image of the number array. Using her own fingers and hands to calculate offers the pupil the opportunity to establish a close identity with material aids. Her consequent independence from extraneous materials is surely the most decisive advantage in the development of her calculating proficiencies. Her own fingers are always there, they cannot get lost and they can be drawn upon for assistance in the most diverse of situations.

By using the fingers and ultimately both of our hands, we link tactile-kinaesthetic, visual and acoustical stimuli; the interplay of senses in turn supports memory storage: not only in the working memory, but also in long-term memory.

Names of fingers

The teacher labels the fingers of her left hand from 1-5. She also labels her pupil’s fingers from 1-5. Should the pupil balk at this, she can wear gloves that display the numbers. The teacher now explains to the pupil that her fingers will be given special names. “This is the first. This is the second, etc.” She also points out to the pupil the exact sequence that has been established, namely the order of the fingers (principle of ordinality).

Why are the fingers labelled?

Individuals with Down Syndrome are frequently visual learners. When they see the numbers on their fingers, they can more easily grasp the connection between the quantity of fingers and the numerals assigned to them. Furthermore, they quickly familiarize themselves with the numerical symbols which, in turn, supports memory storage and recall. Beginners in calculation require a powerful model which they can imitate. In the initial learning stage the teacher therefore counts together with the pupil. If the teacher sits opposite the pupil, she must begin the counting sequence with the little finger of her right hand. If the teacher sits next to the pupil, both begin the counting procedure with the little finger of the left hand.
The counting and calculating exercises introduced in the following scarcely lend themselves to verbalization. The DVD “Yes, we can!” makes things much easier for you, esteemed reader. Part Two of the film, which as a whole deals with the development of calculating proficiencies in individuals with Down Syndrome, is designed as a teaching video. You can directly join in and learn the method through moving images. These, as we all know, express more than a thousand words.

**Counting forward and backwards from 0-5**

The counting procedure begins with two fists, which are placeholders for “zero” or “nought”. The thumb is tucked under the fingers. Both teacher and pupil assume this “starting position”. To the left of their hands is a slip of paper showing the zero. The teacher demonstrates the counting procedure.

In the following presentation the teacher sits next to the pupil. Both fists remain in a relaxed position on the table.

- The left little finger is extended and at the same time “1” is spoken aloud.
- The left ring finger is extended and “2” is spoken aloud.
- The left middle finger is extended and “3” is spoken aloud.
- The left index finger is extended and “4” is spoken aloud.
- The left thumb is extended and “5” is spoken aloud.

Now one counts backwards from 5-0: “That is like during a rocket launch”.

As the next step, teacher and pupil count together from 0-5. That means, they both extend their individual fingers and accompany this with the spoken numbers. Afterwards they count backwards together from 5-0. Even if the right hand takes no part in this, it is essential that it be present: formed as a fist, resting on the table! If the teacher sits opposite the pupil, she begins counting with her right little finger. Important: The pupil always counts from left to right!

Should the pupil initially show difficulty with the refined movements of extending and withdrawing the individual fingers, she may be assisted by the teacher. In this case it would be more practical for the teacher to sit opposite the pupil.
Twins

Together, teacher and pupil search for “twins”.

To begin with, identical:
- What parts of my body are present twice?
- Place 2 identical objects in the open hands
- Look for 2 identical objects in the room
- Tuck 2 pieces of apple inside the cheeks (one piece inside each cheek)
- Lay 2 identical scarves over the shoulders (one scarf over each shoulder)
- Clap, call out, rap, leap, stamp the feet etc. 2 times

Thereafter, similar:
- 2 similar ballpoint pens
- 2 similar plates
- 2 similar forms from the bag of forms etc.

During this activity two fingers are always extended together. Through this the pupil gradually discovers that quantities are hidden within the numbers.

1:1 relationships

“How many are there then?” If we wish to understand amounts quantitatively, we must first take in the whole at a glance but immediately thereafter distinguish the individual parts. If the teacher asks the pupil: “How many apples are there in the basket”, then the first task is to recognize the category “apple” as a group – for there are also bananas and oranges in the fruit basket. As a second step the group “apple” will be reduced to its individual parts. Only then can the pupil begin to count this disorganized quantity; namely, by touching each individual apple with her finger. As the pupil gains experience, she will simply point to the apple without actually touching it.

In order to practice this, let us begin by assigning individual objects to the fingers, following the motto: “1 finger, 1 object”. Now the cardinal aspect of numbers also becomes clear to the pupil: “There are 3 altogether”. Counting must always begin with organized objects, since these can be more easily arranged visually. Only later will the pupil count disorganized groups, such as the above-mentioned apples in the fruit basket.
Counting forward and backwards from 0-10

Now we progress to ten and for this we need the right hand. The teacher labels the fingers to 10. Then she counts with the pupil from 0-10 and back. As soon as this counting process becomes routine, objects will be assigned to the fingers in order to practice 1:1 relationships. It is very important that these objects have an immediate interest for the pupil: E.g. tiny toy figures, familiar everyday items or small treats such as pretzel sticks or pieces of apple. In order to sustain motivation (or perhaps even to awaken it in the first place) unexpected taste treats can accomplish wonders!

Construct stair steps

Using the blue calculating cubes, the yellow number-sticks and the tens-sticks from the “Yes, we can!” calculating kit, one can construct stair steps from 1-10. The pupil closes her eyes and the teacher removes one number-stick. “Which number is missing?” Eyes closed: Two number-sticks are exchanged. “What is wrong here?”

Place your fingers on the number array

The teacher prepares a number array from 0-10. During the counting process, the pupil lays her fingers on the corresponding number of the array. With this exercise the close connection between the array and counting with the fingers is made particularly clear. The foundation for the development of the mental number array is laid. If the pupil is already capable of writing numbers, she herself can prepare the number array.
In addition to counting singly, it is very important to be able to display the correct number of fingers all at once, simultaneously. This is initially managed with quantities of 1, 5 and 10. Thereby important progress is made in understanding the sub-basis 5 and the decimal place 10. The finger image can thus serve as a bridge between quantitative representation, numerical expression and number. In order to practice “lightning quick”, we need a sheet of white paper and a sturdy pencil. The pupil lays her two fists (“zero”) on the paper and the teacher outlines them. Thereafter, on a new sheet of paper, the pupil displays “one”: left little finger extended; the remaining fingers of both hands remain closed (the right fist also rests on the paper). Outline. One after the other, the positions of 0-10 are displayed and outlined. Now the teacher takes one of these 11 sheets and shows it to the pupil, beginning with either 1, 5 or 10. The pupil places the corresponding finger(s) on the sheet of paper, if possible simultaneously, and speaks the number aloud.

A simple box turned on its head (bottom up) and cut out on the longer sides is now colourfully decorated by the pupil herself with paintings or stickers, so that it indeed becomes her own personal box. It is important, however, that she avoid patterns that might later prove distracting.

Teacher and pupil sit opposite each other. Facing each of them is an opening in the box. The pupil places both of her hands under the box. Now she counts from 0-10 and back. In the process, the pupil can no longer rely on her sense of sight and must now learn to trust her sense of touch. She has taken the first step towards developing abstractive powers. If your pupil has difficulties clearly distinguishing her fingers on the smooth table surface, provide her with either a coarse (e.g. burlap) or soft (e.g. felt) cloth covering. Such materials can strengthen the ability to perceive stimuli through the finger tips. If the pupil errs in counting, the teacher may offer assistance.
Which is greater? Which is more?

The ability to place two objects or units opposite each other and to recognize their likenesses or differences is especially meaningful for comparing the sizes of quantities. Relying on finger images, the pupil can see and sense the larger and the smaller number. Through the 1:1 relationship of object to finger, she recognizes the connection between "larger number and more" and "smaller number and fewer". With this exercise it makes particularly good sense to use objects whose sizes clearly demonstrate the distinction between more and less (e.g. bananas).

Subsequent – previous: Number tiles and stair steps

Identifying the subsequent numbers of a series by extending one finger at a time presents the pupil with a small problem of addition. The neighbouring numbers can be recognized quickly if the fingers are labelled and if the pupil can simultaneously interpret finger images (without counting). Identification of the preceding numbers is a great challenge for many pupils. On the one hand, spatial orientation is required ("what comes before, what comes after"); on the other hand, a finger must be retracted. That is a small problem of subtraction that can also be made easier by labelling the fingers.

Puzzles using either number tiles or stair steps marked with small number cards can be amusing. Number tiles may be patches of carpet cut into pieces measuring ca. 20 x 20 cm. Each carpet piece displays one number from the series 1-10. Number tiles can also be made of pasteboard, but in this case beware: Danger of sliding! The teacher displays a finger image for the pupil who in turn takes her position on the corresponding number tile or stair step. She can see the subsequent numbers before her eyes. But what was the previous number? She first controls this by relying on finger images and then takes a step back to check it exactly.
Wherever he may appear, the Number Goblin (for older pupils “Number Anarchist” is perhaps more appropriate) wreaks havoc. He jumbles the order of the number tiles or the cards on the stair steps.

**Now who can restore all of this to order?**
And if he simply turns over some of the number tiles or cards? Which number is hidden on the reverse?

A particularly tricky problem is to select an arbitrary starting point in the number range of 10 and from there to count forward or backwards.

**Here is an example:**
The teacher assigns the pupil the following exercise: “Extend three fingers. Now count forward from the starting point “3”.
The pupil begins with “3” and counts: “3, 4, 5, 6, 7, 8, 9, 10”. Likewise for the return journey: E.g. start with “7” and count backwards. The game “Up and away!” is a perfect preparation for addition and subtraction. The pupil takes up a pre-selected position on the number tiles. The teacher now instructs her to “Take three steps forward/backwards.” “Which number have you reached now?”
Fingers from a different point of view

If the pupil can already simultaneously display the finger images she has learned in the range of 1-10, the time has come to introduce her to further images in other representational forms—e.g., the “three” by extending the thumb, the index finger, and the middle finger. Because this also represents three entities. Using both her hands, the teacher displays various quantities in the range of 10 and the pupil tries to call out the correct number as quickly as possible. Afterwards the pupil displays the quantity using the newly learned finger image. If the pupil already knows numbers, she can likewise point to the corresponding number card, or write down the number herself.

The eyes can also count

Only when the pupil is in sure control of her counting skills should she let her eyes do the counting: That is, to count without allowing her fingers to participate.

Here is an example:
If the teacher poses the question “How many cars do you see in this parking lot”, then the pupil must carefully sort through the categories. In this parking lot there are plants (trees), people and vehicles (bicycles and cars). Only the disordered group “cars” will be reduced to its single elements, with the eyes spatially grouped and then counted. Not at all simple. Absolutely start with small quantities!
Always the same!

Understanding invariance
Daniela is happy: Her ice cream cost 80 Cent. She paid for it with a €2 coin and got back even “more”. She proudly shows her mother the 6 coins in change, which together amount to only €1.20. Daniela sees this as a good bargain: Give up one coin, receive 6 in return, and an ice cream as an added bonus. Daniela lives with that certain Extra and is only 7 years old. She is in need of a great deal more experience with respect to “learning by doing”, which will slowly bring her closer to understanding invariance.

**What does this term mean?**

Invariance is understood to be the constancy of a quantity, even if this quantity has changed its appearance. An example: A ¼ litre of water is poured from a glass into a pitcher. There it gives the illusion of being less water than in the glass, but it is nevertheless still a ¼ litre of water. Through multiple experiments in a variety of everyday situations, the pupil gradually learns to recognize that a given quantity remains the same even if it has changed its shape, perhaps its height or dimensions. Purely structural alterations by means of rearrangement, division, redistribution or separation of objects in a row nevertheless leave the total quantity unchanged.

![Apples, bread and pizza](image)

The kitchen is a perfect setting for experimenting with invariance! By cutting up apples, oranges, bananas, slices of bread, cake, pizza etc. in various ways and consciously reuniting their parts, individuals with Down Syndrome develop a self-motivation that is for the most part so great that the desired learning effects evolve of themselves. Thus: Learning with “head, heart and hands”, as the Swiss pedagogue Heinrich Pestalozzi urged more than 200 years ago.
Fold paper

The pupil takes 2 identical sheets of white letter-size paper. He crumples or folds one of these numerous times. Afterwards he unfolds it and compares it to the second, smooth sheet.

Form a queue, please!

Working with children: The teacher takes perhaps 5 toy men and arranges them haphazardly. The pupil counts the figures and notes the total. The teacher now explains that these men are waiting for the zoo to open. When the ticket-booth finally opens, the men politely form a queue. The teacher queries the pupil: “How many men are waiting in the queue? If the pupil counts the figures anew, his understanding of invariance with respect to quantity is still not sufficiently developed.

Working with adolescents and adults: One can perhaps use toothpicks to represent the people who are waiting for the cinema box-office to open.
Cubes or stones, chestnuts, potatoes

Two rows of 5 cubes each are placed on the table, maintaining an equal distance between the cubes.

The pupil counts the upper and lower rows separately from one another.

The pupil is asked:
“How many cubes are above?” “How many cubes are below?”

It is important to observe that the pupil knows the exact quantity in the upper and the lower row and can state this aloud.

Now the teacher spreads out the cubes in the lower row.

The pupil is asked:
“Do both rows have the same number of cubes, or does one have more?”

If the pupil now begins to recount both rows, the teacher realizes that sufficient understanding of invariance is lacking. A pupil who has already developed this understanding will no longer count the rows but will state with certainty “They have the same.”

Afterwards the teacher uses the five cubes to form a tower, a circle, a row, a random display etc. The objective is for the student to discern that the total quantity remains unchanged despite rearrangement of its parts. The pupil should be afforded many opportunities to experiment with one and the same number of cubes. Count- rearrange- count- rearrange.
The pupil fills two transparent water bottles with equal amounts of water, approximately half full. He then closes the bottles tightly, places them on the table and compares them. The pupil explains that both bottles contain the same amount of water. While he watches, one water bottle is inverted—thereby the water level rises although the total quantity remains the same. Now the teacher asks the pupil the following question: “Is there the same amount of water in both bottles, or does one contain more?”

When repeating this experiment at a later date, it is recommended to reverse the order of the question: “Is there more water in one of the bottles, or are the amounts equal?” It is very important always to exchange these two questions during the various experiments because individuals with Down Syndrome frequently tend to repeat as answer the last thing they have heard. If the pupil answers “That one has more”, the teacher continues to query:

• “Did you add water to one of the bottles?”
  (wait for the answer)
• “Did you remove water from one of the bottles?”
  (wait for the answer)

Afterwards teacher and pupil together return the inverted bottle to its original position. The two bottles are compared to each other; the pupil confirms that they are now once more equal.

It is essential that the teacher never reveal the correct answer!

If the pupil is provided with the correct answer, then during the next experiment he will most likely say “they are alike” without ever having achieved an understanding of invariance. If the pupil’s answer is correct (“they are alike”), he should also not be overly praised or too greatly encouraged, rather the teacher should simply repeat his answer. He has not really accomplished anything remarkable, rather he has simply progressed one step further in his natural cognitive development.
Two glass containers are readied for use: one tall and narrow, the other low and wide. The pupil prepares two equal quantities of sand. He pours the one amount into the tall, narrow container and the other into the low, wide container.

**The teacher queries:**
“Is there the same amount of sand in both containers or does one hold more?” (“Is there more sand in one of the containers, or are the amounts equal?”) If the answer is incorrect, the teacher proceeds:

- “Did you add sand to one of the containers?” (wait for the answer)
- “Did you remove sand from one of the containers?” (wait for the answer)

Multiple, self-performed experiments involving shifting quantities of sand, marbles, small pieces of gravel, water etc. back and forth aid the pupil in developing an understanding of invariance.

Modelling clay

Modelling clay is used to create two large balls identical in size. These are placed side by side and compared. Once the pupil has agreed that both balls (or “dumplings”) are the same size, the teacher uses one of the dumplings to make a pancake.

**The pupil is asked:**
“Is there the same amount of modelling clay in each or does one have more?” (“Is there more modelling clay in one of the objects, or are the amounts equal?”)

**If the answer is incorrect:**
- “Did you add clay to one?”
- “Did you remove clay from one?”

Calculating with fractions

The ability to calculate with fractions derives from an understanding of the principle of reversible alteration. Objects that are first divided and then returned to their original state offer abundant opportunities to observe invariance. How can the teacher explain the fraction stroke as symbol? Very easy! He places a slice of bread on a plate and cuts it with the knife horizontally into two equal slices. He makes a fraction stroke out of cardboard, places this between the two half-slices and explains to the pupil: “This is where I made the cut. These are now two half-slices of the original piece of bread.” Using small cards (see the worksheet), the pupil is introduced to the fraction \( \frac{1}{2} \) and how to write it.

Other food products that can be divided horizontally are used to introduce further fractions. Liquids, e.g. a \( \frac{1}{4} \) litre of water, also lend themselves exceptionally well to experimentation. As a final step the various fractions are compared to each other.
Working with numbers paves the way to the student’s understanding of symbolic, abstract mathematics. They introduce structure into counting procedures. The number is written down, and is therefore not only acoustically perceptible – as when counting aloud – but also, above all, visually perceptible. Larger quantities in particular are summed up in the numerical symbol and, as a result of this, more easily comprehended. Numbers are primarily memory aids that help us retain information. All of the exercises in this book that concern themselves with counting, writing numbers and calculating can be supplemented with the wooden numbers from the calculating kit. These are valuable not only for fingering and touching when the eyes are closed, but also as a support for counting and calculating procedures.

**Line patterns**

The teacher draws a line pattern on a sheet of paper (in the beginning just three lines, later on a greater number). The pupil copies this example (from memory). Various variations for copying: In addition to drawing on paper, there are numerous other possibilities for drawing using all of the senses – in a sandbox, with a few drops of oil on a baking sheet, with water colours on a sheet of wet paper, with shaving cream on a mirror, and many more.
Round like a cup
Household items (e.g. a cup, a clock, a cushion) are sorted according to their form and then accordingly labelled: “round like a cup, rectangular like a cushion, triangular like the (cold) surface of an iron” etc. Each player takes a form in his hand and feels it, i.e. examines it with his hands, to determine the structure (round, rectangular, triangular). The teacher gives directions: All circles proceed to the door, all rectangles sit down, all triangles raise both hands.

From the circle to the figure eight
Before a pupil learns to write numbers, he must himself be able to draw simple forms. These include the circle, horizontal and vertical lines, their combination as crosses, rectangles, triangles, and finally their recombination to create houses and the like. Loop motions are required to form the figure eight. In addition to being able to write numbers, it is important that the pupil recognize numbers in a variety of forms: In his own handwriting, in notes jotted down by others, as three-dimensional figures created from various materials (wood, modelling clay, dough etc.), from various computer fonts, as well as through various media, whether PC, pocket calculator or mobile/cell phone.
Yes we can!

**Back “mail”**

Draw simple forms, such as lines, circles, rectangles and triangles, on a sheet of paper and place this on the table. The pupil now receives a communication (“mail”) from his back. That is, his playing partner, observing the forms on the table in front of him, draws one of the forms on the pupil’s back. As soon as the pupil recognizes it, he points to it on the paper. This exercise is more difficult with numbers written down on paper and also drawn on the pupil’s back. Wooden numbers are placed before the pupil. When he recognizes a written or drawn number, he should point to the corresponding wooden number. It becomes very difficult when the teacher firmly but gently applies pressure with a certain number of fingers, between 1 and 5, on the pupil’s back. The pupil indicates what he has sensed by pointing to the corresponding wooden figure (or by displaying the correct number of fingers). Or: In front of him are dot pictures, and he points to the picture that displays the correct number of dots.

**Fingers, dots, numbers**

**Lotto**

Dot cards (with both structured dice patterns and random patterns) are matched to numbers. The pupil carefully compares the dot cards and places the correct number card on the dot card.

**Eagle eyes**

Now the teacher displays a number of fingers, e.g. four. The pupil should select the corresponding dot card and/or number card.
Joker

All of the dot cards and the Joker card are distributed among the players. Each player hides his cards in his hand. There is no card stack in the middle. Now the players take turns drawing from each others’ hands. If a player has a matching pair in his hand (e.g. two different cards that each has three dots), he may discard this pair. Each player tries to draw the Joker from the other player’s hand. The winner is the one who in the end has the Joker in his hand.

All 6!

The inverted top of a shoe box is used as a bowling lane. It is placed upside-down on the table. Positioned at one end are 6 small, toy bowling pins numbered from 1-6. A marble is rolled down the bowling lane. Which pins does it strike? The numbers of the toppled pins are matched to the dot cards, and subsequently not only the wooden numbers, but also the corresponding number of fingers, are displayed.

Who can topple 6 pins?
Who can not only roll the marble, but also shoot it with his finger?
Calculating develops from counting. Our 10 fingers offer the perfect support for representing quantities within the decimal system. By counting further from a particular starting point (in the game “Up and away!”), one has already solved the first problems of addition and subtraction in the number range of 1 to 10.

Here, as in the chapter “Counting to ten”, explaining games and exercises in written form can often become overly involved. Please watch the instructional video on the DVD “Yes, we can!” All will then be clear!
Addition and rubber bands

In order to demonstrate that addition consists of two partial quantities (the so-called addends), two rubber bands can be called into play. One is used to bind together the first partial quantity; the other, the second partial quantity. The sum, a cardinal number representing the final quantity, is recognizable at a glance. At this stage the addition lies in written form before the pupil. As the first step in the learning process, the teacher offers the pupil a model for imitation: She calculates the addition alone and recites accordingly: E.g., “4 and 3 equal 7”. As the second step, teacher and pupil calculate and recite together. Once the pupil has come to understand that the addition is comprised of two partial quantities, the rubber bands can be put aside. Pupils who balk at using rubber bands can work with a plus-sign cut out of pasteboard and placed between the fingers or finger-groups representing the partial quantities.

• Initially the partial quantities are displayed counting individually.
• In order to develop abstract thinking it is crucial to learn to display the partial quantities simultaneously. The pupil displays the first partial quantity; the second is initially displayed counting individually, but with sufficient experience it can also be displayed simultaneously.
• Once the process of addition has become routine, it is important to execute it under the secret box. If necessary, the teacher can offer assistance.
• For calculations that require only the left hand (e.g. “3+2”), both hands should nevertheless be placed on the table- the right hand as a fist.
• For the calculating process to succeed, it is extremely important that all steps of the calculation be spoken aloud! Brain research explains why: The cross-linkage between finger calculations and their aural verbalization forms a critical connection between the representational forms of numbers in our brains. Examinations show that the language areas of the brain control exact reckoning, whereas the spatial areas, together with those responsible for finger activity, control approximations.
Subtraction and rubber bands

Subtraction in the number range of 1 to 10 proceeds analogously to addition. The first partial quantity (the minuend) is initially displayed counting individually. Afterwards the second partial quantity (the subtrahend) is bound together with a rubber band (or set off with a long minus-sign of pasteboard). The result is again recognizable at a glance. After the pupil has gained sufficient experience, the partial quantities can be displayed simultaneously, and the secret box can also be put to use again.

Reducing numbers

In order to prepare for additions and subtractions that cross between the tens-boundaries (or bridge through ten), the pupil reduces or splits numbers in a variety of ways.

Taking “5” as an example, several possibilities are demonstrated:

- 5 fingers are displayed as a unity. The teacher discusses with the pupil the possibilities for reducing the number: 5 is reduced to 1 + 4, to 2 + 3, to 3 + 2, to 4 + 1. The individual partial quantities displayed by the fingers are separated by the pasteboard plus-sign.
- 5 fingers are displayed as a unity. A small blue cube is assigned to each finger. Now the pupil closes her eyes and the teacher removes two cubes. How many are missing? Afterwards teacher and pupil discuss number reduction: “3 and 2 equal 5”.
- Teacher and pupil agree upon a whole quantity, e.g. “5”. Now the teacher slowly drops 4 small blue cubes one after the other onto a porcelain plate. The pupil counts each individual cube aloud and also displays the corresponding number of fingers. 4 cubes lie on the plate, but there should be 5 altogether. How many cubes are missing? The one finger that remains tucked into the hand helps to solve this challenging supplementary calculation, also known as “and-how-many calculation”.
- Part-whole: After having reduced the numbers, the pupil must now restore the partial quantities to a whole.
Tens-friends

Tens-friends are an exceptional team because they supplement each other perfectly. Who are these friends then? The nine and the one, the eight and the two, the seven and the three, the six and the four. Only the five has chosen his twin as best friend. A picture representing each of the two tens-friends can prove to be an easily-remembered memory aid.

Numbers to 20

Exchange

For calculating up to 20 the fingers now require an additional aid: This is the red 10s-stick. Red, because the tens-column in many accounting notebooks is indicated by the colour red. If that is not the case in the notebooks you use, please repaint the tens-stick using a matching colour. So that the pupil understands that a tens-stick indicates a new place value and that it can also be exchanged for the 10 fingers, we must draw upon our knowledge of 1:1 relationships.

• The pupil displays 10 fingers and the teacher assigns a blue ones-cube to each finger. Many a young girl loves to have “blue fingernails”: Therefore a piece of tape is used to attach a cube to each fingernail. Finger games with the blue nails clearly heighten the ability to grasp that finger and cube belong together. Now the pupil places all of the ones-cubes together below a tens-stick. She thereby actively sees that 10 ones-cubes (therefore 10 fingers) can be exchanged for a tens-stick.
• This tens-stick now represents the 10 fingers.
Count to 20

The starting position for counting and calculating with the tens-stick lies in the two fists (important: the thumbs are tucked under the fingers). The tens-stick is placed horizontally above the fists.

- The pupil now counts from 0 to 10, speaks aloud “exchange”, and positions the tens-stick vertically to the left of her two hands.
- Now she retracts her fingers; her two fists are positioned to the right of the tens-stick.
- The counting process resumes. “11, 12 … to 20”.
- Having arrived at 20, the pupil raps on the table and begins counting backwards.
- When she arrives at 10, a tens-stick and two fists lie on the table.
- Now again “exchange”.
- The tens-stick is returned to the horizontal position above the two hands and the fingers of both hands are now extended.
- Now she counts backwards from 10 to 0.
Yes we can!
Calculating through 20 proceeds logically from adding and subtracting through 10. The only difference is the use of the tens-stick. Particularly in this number range many pupils quite easily recognize the analogies of the decimal system.

“2 and 4 equal 6”
“12 and 4 equal 16”

During these calculations the tens-stick is placed at the left of the two hands, and the fingers, precisely as when calculating through 10, represent the ones.

“7 minus 5 equals 2”
“17 minus 5 equals 12”

If the calculations are written down, the tens are more easily recognizable if entered in red. As when counting through 10, partial quantities are initially displayed counting individually, but with sufficient experience they can also be displayed simultaneously. Calculating under the secret box is of great importance in this number range! And you already know: Please accompany every step of the calculation reciting aloud.
Crossing the tens-boundaries

Only after the pupil can ably add and subtract through 10 and 20 should the teacher begin to work with her on crossing the tens-boundaries – for this is tricky and particularly challenging for many students with 46 and 47 chromosomes. The most important prerequisites for success in adding and subtracting between the tens-groups are the experiences gained in counting with the fingers and in reducing numbers, as well as knowledge of the tens-friends.

Teacher and pupil sit opposite each other. An addition that crosses between the tens-groups is written out on a card and placed before the pupil.

An example: $9 + 4 = \phantom{0}13$

The starting position is the basic position: Two fists, and placed horizontally above them, the tens-stick.

Now the pupil displays the first partial quantity, namely, 9. The teacher taps the one finger that is still tucked in and asks: “Do you still have 4?” The pupil answers: “No, 1”. The pupil counts further: “1”. This followed by: “Exchange”. She places the tens-stick to the left of her two hands and forms fists. Again she continues counting: “2, 3, 4”. With a glance she sees the result, namely, 13. The following problem can arise AFTER the exchange: The pupil has forgotten how many additional fingers she had displayed BEFORE the exchange. In this case, it was one finger. Now the teacher assumes the role of the so-called “extraneous memory”. After the exchange she repeats the number “1” (representing the finger that had already been added), and the pupil continues with “2, 3, 4”.

Once she has gained experience through practice (a period of time that can vary in length, depending on the individual), the pupil assumes the role of teacher. She reminds herself of the quantity and whispers this when making the exchange. This aural accompaniment should gradually be replaced by an inner voice, an ability to think for oneself. The same principles apply for subtraction that crosses between the tens-groups.

An example: $15 - 9 = \phantom{0}6$

For this subtraction the pupil first displays 15. Then the teacher asks her the question: “Do you have 9?” The ensuing procedures are oriented to the previously demonstrated addition. With respect to crossing the tens-boundaries: Both calculating under the secret box and the simultaneous display of partial quantities are significant stepping stones towards abstract thinking. The simultaneous display of the second partial quantity demands a solid knowledge of the technique of reducing numbers!

In the video “Yes, we can!” you will find multiple examples for joining in.
Measure with the tens-stick

The tens-stick is exactly 10 cm long and is optimally and variously suited for becoming better acquainted with this dimension (1 dkm). For one thing, the pupil often has the stick in her hand and can therefore compare it to her finger span. This finger span should also be shown repeatedly, without the stick, and used to approximate measurements in everyday situations. “Is this book longer or shorter than the tens-stick? What do you think?” Now the intuitive finger span is first drawn upon, afterwards the actual tens-stick.

Incidentally: The blue ones-cubes measure 1 cm along the edges, and the number-sticks from 2 to 9 have a corresponding cm length.

Numbers to 100

Analogy calculations

Navigating to 100 is learned through repeated application. The knowledge gained doing calculations through 20 is now expanded and applied in a variety of ways. The initial approach to 100 begins with counting to 30, by the introduction of another tens-stick, and carried through to 50 in similar fashion. At this stage the red fifties-board can be introduced: It is the same exact size as 5 tens-sticks placed one against the other. This continues step by step up to 100.

The structure of the decimal system allows the pupil to solve addition and subtraction problems by means of analogy.

An example of addition:

- \( 5 + 3 = 8 \) (calculated with the fingers)
- \( 15 + 3 = 18 \) (calculated with the fingers and one tens-stick)
- \( 45 + 3 = 48 \) (calculated with the fingers and four tens-sticks)

At the start of the calculation the requisite tens-sticks always lie horizontally above the fists. When called for during the calculation they are placed vertically to the left of the two hands. Please keep in mind: For the calculation process to succeed, it is vital that every step be accompanied aurally! And calculating under the secret box supports the development of abstract thinking.
Partial tens

While both subtraction and crossing of the tens-boundaries through 100 function according to the same exact principle as calculating to 20, calculating with partial tens presents a further challenge to the pupil.

An example:
\[ 25 + 34 = \]

10 tens-sticks lie horizontally above the fists.
The pupil places two tens-sticks to the left of her two hands and speaks aloud: “10, 20”. She then displays the five fingers of her left hand and speaks aloud: “25”.
Now she first adds 30 by placing three more tens-sticks to the left. Altogether there are now 5 tens-sticks. The pupil speaks aloud: “55”. As the last step she displays another four fingers on the right hand and reports the result: “59”.

Addition of partial tens that cross the tens-boundaries – e.g. 25 + 39 – and subtraction function according to the exact same principle. You know it anyway: Accompany the calculations aurally and also calculate under the secret box!

The DVD “Yes, we can!” offers a number of handy examples of partial tens.

“The other way around”: The problem of the ten-one inversion

The German language is burdened by the problem that within the 100-range the tens and the ones are inverted when spoken. For 63 we thus say “three-and-sixty”. Here we must learn an important rule: Within the 100-range we must first speak the ones (= the fingers)! Thereafter come the tens. A mnemonic aid: “The fingers say ‘hello’ (signal with the fingers). This sentence contains the key words “finger” and “say”. When writing, of course, it is entirely different. Now comes the second important rule: We write down the numbers just as we see them displayed before us, with sticks and fingers. That is, to the left the tens and to the right the ones. Another memory aid: “Write it as I see it”.

What pleases is permitted …

If the pupil feels quite secure using the tens-sticks and her fingers for calculating, she can now gradually begin to replace these with everyday materials: Instead of the tens-sticks, coloured pencils or straws, cutlery in restaurants, or small sticks and stones along a path. This everyday material encourages her to think in more general terms, and anything that pleases is permitted. The use of €10 bills is particularly relevant. It is important only in the beginning to limit the materials so that mental imagery can be allowed to develop. Afterwards, however, it can be quite amusing to experiment with the materials.
Tens-knuckles

During the long journey towards abstract thinking, we must first create a link between the concrete concepts of quantity and number. Illustrations with material aids contribute to the meshing of these two areas of knowledge. Temporarily calculating under the secret box both gradually and gently weans dependence upon visual impressions. We take a decisive step towards gaining independence from extraneous calculating aids when we learn to use our own knuckles to represent the tens. We begin by labelling these with "10, 20, 30 … to 100" (either writing the numbers directly on the knuckles with an ink pen or pasting small numbered stickers on the knuckles). When crossing the tens-boundaries, “exchange” corresponds to “spring” from one tens-knuckle to another, the latter representing either a greater or lesser quantity. 2 hands and 10 fingers are now capable of adding and subtracting from 0-100!

Please watch the DVD “Yes, we can!” and join in. You will be surprised at the ease and efficiency of using the tens-knuckles.

Hundreds-boards – hundreds-lines

The use of calculating materials progresses consistently to 100 and above. The green board that symbolizes the hundreds is the same size as 10 tens-sticks. Green, because most accounting notebooks use that colour to represent the hundreds. If your books are different, you must repaint the board the matching colour. We can dispense with extraneous materials within this range of numbers as well. The hundreds are drawn as a line on the back of the hand, beginning below the tens-knuckles. When calculating, always point to the corresponding line.

An example being the representation of 174:

We begin by drawing the hundreds-line on the back of the hand (below the knuckle of the first finger). For 70 we touch the seventh tens-knuckle (below the seventh finger). Finally, we extend 4 fingers. This significantly challenges both the memory and the capacity to think abstractly! Calculations of amounts exceeding the hundreds are dealt with by individuals with that certain Extra just as they are dealt with by anyone else: By writing them down. In this way large numbers suddenly become small because they can be grouped and exchanged according to individual place value.
Multiplication with actual objects

“Times and plus belong together”: Multiplication grows out of addition. 20 carpet tiles are laid out and the pupil steps onto the second tile. In the process she speaks aloud: “1 times 2 equals 2”. Afterwards the pupil steps to the fourth tile and says: “2 times 2 equals 4”. She progresses through the multiplication table to 20 and then returns to 0. Working with real objects establishes a link not only to addition, but also to division.

An example: \[2 + 2 + 2 = 6\] \[\times 2 = 6\]

Which actual objects could we use? Here a few suggestions:

- For the multiplication table of 2: cherries or apple slices
- For the multiplication table of 3: keys
- For the multiplication table of 4: toy cars (wheels)
- For the multiplication table of 5: pencils
- For the multiplication table of 6: leaves of a tree/shrub on a single branch
- For the multiplication table of 7: stones
- For the multiplication table of 8: pretzel sticks
- For the multiplication table of 9: grapes
- For the multiplication table of 10: pictures of the 10 fingers

Working example for the multiplication table of 7:
The pupil collects 70 small stones while taking a walk. She places seven stones each on different plates. Next to each plate is a blank card. On the first card the pupil writes: “1 • 7 = 7”. She comes to the second plate and calculates (with the aid of her fingers): “7 + 7 = 14”. She writes on the card: “2 • 7 = 14”. Following this method, the multiplication tables can be built up gradually through “5 • 7”, or one can skip immediately to “10 • 7”.

Along with addition it is also important to provide the pupil with fundamental experience concerning “what is contained therein”. This lays the foundation for division.

“You have a total of 21 stones. Look, they are divided among three plates. Each plate contains 7 stones. 7 is contained exactly 3 times in the number 21.” Between these explanations the teacher must pause. Afterwards the teacher queries the pupil about the facts of the matter: “How many stones do we have altogether?” “How many plates do we have here?” “How many stones are in each plate?”

Occupying oneself frequently and in concrete terms with actual material can gradually lead to the understanding of division.
To improve memory storage, the multiplication table for 2 is additionally demonstrated by fingers bound in pairs and one tens-stick. Successful instruction again depends on a verbal accompaniment spoken aloud. From the multiplication table of 2 the pupil can easily recognize that multiplication in general proceeds from the bundling of quantities (fingers). Most pupils memorize the multiplication tables. To simplify this step, it can be advantageous to employ a technique learned in memory training: the so-called loci method. In this method each multiplication table is assigned to a specific part of the body.

In the video “Yes, we can!” you will find the following suggestions:
- Multiplication table of 2: the face
- Multiplication table of 3: the right arm
- Multiplication table of 4: the left arm
- Multiplication table of 5: the inside of the right hand
- Multiplication table of 6: the inside of the left hand
- Multiplication table of 7: the right leg
- Multiplication table of 8: the left leg
- Multiplication table of 9: the stomach area
- Multiplication table of 10: the tens-knuckles

Each individual multiplication unit within a table is assigned to a specific point on the pre-selected body part, and this point will be touched when reciting the multiplication (and its result). This establishes a strong connection between a particular point on the body and a particular multiplication unit. These so-called anchors of association facilitate storage in both working memory and long-term memory. When repeating the multiplication tables, the pupil first generally touches the representative point on the body and thereafter speaks the result aloud.

The prerequisite for solving division is a complex succession of multiple calculating steps from various fundamental types of figuring and reckoning. The body positions selected for multiplication can also be used for the so-called “into-calculations”. They provide the basis for performing division. Since the approach to division often differs from country to country, we can offer only a few general tips here. As preparation for simple division it is advisable to write down the necessary multiplication table. This visual prop helps to reduce the demands on the working memory. One could also present the information in tabular form, in order to show at a glance how often the smaller number is contained in the larger. This could be set off by a line between the various results. Pros already rely on the body positions. “How often is 3 contained in 15?” Reciting the multiplication table and simultaneously touching the corresponding position for the multiplication table (e.g. a point on the right hand) already reveals “what is contained therein”.
Factual calculations

“To teach details is to foster confusion. To establish the relationship between things is to sow knowledge” (Montessori). Our everyday math is full of factual calculations involving addition, subtraction, multiplication and division. In order to motivate the pupil to meet these challenges, the most important thing of all is to relate everything to her own personal life. That should mean not having her solve anonymous problems from a math book, rather, her own individual requirements (as they arise daily) when shopping or counting her change, when examining a so-called special offer, when dividing up a desert or perhaps when calculating her own energy reserves (see also the chapter “Life as it is”).

Estimates and rough calculations

Brain research has taught us that our ability to estimate and our ability to do exact calculations are controlled by two different areas of the brain. Being able to calculate with exactness unfortunately does not necessarily lead to being able to estimate with assurance. For many of life’s situations the pupil should therefore be animated to learn how to estimate. “How many books do you think there are in this bookcase?” If the teacher is confronted by a puzzled expression on the face of the pupil, or if she hears a number that is wide of the mark, she can suggest possible ranges: “Do you think there are about 10? Or could it perhaps be 50? Or are there about 100 books in the bookcase?” After an initial estimate, teacher and pupil together count the books.

A mathematical chapter that falls in the area of comparison is the determination of quantity dependent upon context. What is concerned here is recognition of the fact that, for instance, the quantity 10 may in one situation be “many”, in another situation “few”. 10 spectators at a football game are few, but 10 guests in my living room are many.
The instantaneous recognition of organized dots can most easily be practiced with dice. Dice faces are “lightning images”: that is, the quantity of dots in cubic form should be grasped at a glance and verbally identified (and/or displayed with the fingers). The opportunities for creating one or several fanciful dice games are all but unlimited. Since dice can in many ways be highly motivating, mathematical instruction can scarcely be imagined without them. In case of “drooping” on the part of the pupil (or the teacher), or as “ice breaker” at the beginning of a math session, or as “reward” after a cooperative effort: Playing with dice is simply fun! The larger the cube, the greater the amusement, inasmuch as large cubes encourage participation with the entire body and thus provide ever new impulses for counting and calculating. Here follows a brief compilation of a few dice games, from simple to tricky, that have proved themselves again and again in our work with individuals with Down Syndrome.

Clap 4 times and hop 1 time

The pupil rolls the die and in accordance with the number of dots on the face performs the appropriate number of pre-arranged motions: e.g. clapping, hopping, running around the die etc.
Number Goblin II, or Number Anarchist

Number tiles arranged from 1-6 lie on the floor. The pupil rolls the die and then searches for the tile that matches the number on the face of the die. She then places the correct number of objects on the number tile. All of the tiles from 1-6 are covered in this way (if necessary, the number range can also be reduced).

Afterwards the rule is once again “Eyes closed!” because the Number Goblin is underway. As always, he wreaks havoc and makes a jumble of the objects on the tiles. Can the pupil restore order?

Dice football

Two girls play football with a foam rubber die. If a goal is shot, the number of dots atop the face of the die determines the score.

Who wins which round?
If the girls already know how to do addition, they can add up the number of dots from the different goals. Who has won the most games in all?

Cubes on dice

The pupil rolls the die, identifies aloud the number of dots, and places on each dot a small, blue ones-cube (from the calculating kit). Afterwards she closes her eyes and her playing partner removes one or more of the blue cubes. Eyes open! “How many are missing?” A wonderful complement to reducing numbers!
Yes we can!

Dots everywhere

On the worksheet in the appendix you will find cards with disorganized and organized quantities of dots.

- These are cut out and the pupil matches them up – according to the number of dots.
- Who has the most? All of the cards are distributed among the players. Each player turns over her top card. The player whose card has the most dots takes all of the other cards. In case of a tie, turn over the next card.

Build houses

The pupil rolls the die and notes the number of dots. This is her personal number for the remainder of the game. Her playing partners are also assigned their personal numbers by a roll of the die. Each time a pupil rolls her own number in the course of the game she is allowed to draw one line of a pre-selected figure (e.g. house, car, ship) on a sheet of paper. The first to complete the figure is the winner.

Tip-tap

The pupil rolls the die and taps out the number of dots on the back of her playing partner (or if her partner’s eyes are closed, on the back of her hand or thigh etc.). Her partner counts along, speaks the number aloud and compares her results with the die.

Let it be

The pupil rolls as long as she wishes to, and is allowed to add up all of the points together. But beware: As soon as she rolls a six, she loses all of the points from this round and her partner takes a turn. Who is the first to win 30 points?
Lightning calculator

2 dice are placed in a cup. The pupil and her playing partner take turns rolling, each one a total of 10 times. In front of each player is a slip of paper with 5 plus signs and 5 minus signs. These must be used up during the 10 rolls. Before rolling, the pupil must decide if she wants to calculate using a plus or a minus sign. The corresponding sign is crossed out on the slip of paper. Then she rolls and tallies the result. She writes this down on the slip of paper. Now it is her partner’s turn. After 10 rolls all of the results are tallied. Who has the most points?

Roll 12!

The pupil and her playing partner each prepare their own set of playing cards with the numbers 1-12. Each card bears one number. These cards are placed in an open stack in front of each playing partner, with the 1 on top and 12 on the bottom. Now one of the partners rolls three dice at once.

- The goal is to eliminate the 1-card. How does this work? Either a die displays 1 dot or 1 can be arrived at through addition or subtraction (e.g. 1 die displays 4 dots, the other 3 dots: 4-3=1) Either two or three dice are used for calculations, and addition and subtraction can be merrily combined.
- If a roll makes it possible to eliminate several cards one after the other, this is not only allowed, but also desirable!

**One example:**
The dice display 5 and 3 dots, as well as 1 dot.
Now we eliminate:
Card 1 1 dot) ( d
Card 2 5-3) ( d
Card 3 3 dots) ( d
Card 4 5-1) ( d
Card 5 5 dots) ( d
Card 6 (5+1)

- If no cards can be eliminated with a roll, the other player takes her turn.

The first to eliminate all of her cards is the winner!
Day-to-day mathematics!

The alarm rings at 6 AM. Daniel, 16 years old and born with that certain Extra, begins his day with numbers. Sleepily he counts the days until the week-end – his salvation – when he can sleep to his heart’s content. In the bathroom Daniel has to apply just the right amount of toothpaste to his toothbrush. Later, at the breakfast table, he divides a litre of tea among four cups that hold 250 ml apiece. He cuts his breakfast bread in half and spreads it with marmalade. How much does he need for each slice? In his sock drawer Daniel finds a matching pair, but unfortunately his favourite pants are now not only too short, but also too tight. Has he perhaps grown? On the wall is a measuring strip where he can check his height. And indeed, 4 cm taller. A quick glance at the clock tells him that his bus will be arriving in 5 minutes. Now he must hop to it! In the lift he presses zero for the ground floor, and as he is greeted by the sun as he steps outdoors, he knows: Spring has finally arrived! Not much has happened yet today, but still, Daniel has been confronted with numbers at every turn: keeping a check on the clock, comparing quantities, taking measurements, thinking about the days of the week and the seasons of the year. After school Daniel still has to do some shopping and go to his dancing lesson, and on the week-end he wants to take in a movie. Hopefully he will still have enough money for this! Whether money or the rhythm of music: Numbers surround us. The motivation to busy ourselves with numbers grows as we learn to identify them with our personal lives. For this reason, teaching must also be oriented to life. When, in every-day situations, do I look at the clock, where must I arrive punctually? How much time do I need for each activity? When do I reach for the measuring stick, the scale or the calendar? And above all, how often do I reach for my pocketbook? Is there enough money in it for all I have to do? Adults with Down Syndrome who are in charge of their own money can benefit by having their own bank account, one protected against overdraft, but providing them with a bank or ATM card. This allows them to play an active role in shopping, even in situations where they might not be totally prepared for quick exchanges of monies (like a busy check-out counter). Using a PIN code also keeps their long-term memory fit. And they learn to keep a secret.
Money

In the following exercises the Euro is to be understood as representative of all currencies.

EURO 1 Notebook

In order to learn to manage money, it is important to use real money! Only if the teacher orients herself as closely as possible to the actual life of her pupil will the pupil be able to understand the connection to her own daily existence. From the age of 6, children should be provided with a bit of pocket-money. A small coin, e.g. €1, with which they may do as they please, quickly enables them to learn how much things cost. In a small notebook one can paste in pictures of objects or food items that each cost approximately one Euro. Later one can create notebooks for €2, €5, €10 etc. There are certainly enough advertising flyers floating about the house daily that are ideal for making price comparisons.

Money cups

Prices from advertisements are pasted on the cups.
Prices of approximately €1 are pasted on the first cup.
Prices of approximately €2 are pasted on the second cup.
Prices of approximately €5 are pasted on the third cup.
Prices of approximately €10 are pasted on the fourth cup.

Pupils who are already able to calculate in higher number ranges can label additional cups with prices of approximately €20, €50 and €100. Now the pupil cuts out the pictures of products from various flyers and places these, according to price, in the appropriate cup. In the process, she must constantly round prices up or down. As the next step, all of the pictures will be removed from the cups and shuffled. Which picture now belongs to which cup?

Fingers and money

Depending on the range of numbers the pupil uses for counting and calculating, one should expand the activity by introducing bills and coins. The €1 coin should be assigned to the ones-finger, the €2 coin to the twos-finger etc. Slowly, through daily practice, the pupil will learn to identify the value of coins and bills.
Euro, comma, cent

Cent pieces should not be introduced until the pupil has learned to calculate through 100. First the pupil must learn to distinguish between Euros and cents. Afterwards one assigns to the ones-finger 100 cents in various combinations (e.g. $2 \times 50$ cents or $3 \times 20$ cents plus $4 \times 10$ cents etc.). To indicate prices that combine Euros and cents, it is advantageous to select a specific method for indicating the comma (= decimal point). Since Euros are primarily silver-coloured, they should initially be written with a lead pencil. Cents, which are copper- or brass-coloured, are correspondingly written with a brown coloured-pencil. For the comma another colour should be chosen, perhaps violet. This marks the division between Euros and cents.

The bill, please!

Bills from restaurants or shops are saved and used to practice rounding off amounts. The pupil first searches out the total amount (e.g. €24,30) and thereafter rounds this up (to €25,-) in order to prepare for the actual situation of paying. The appropriate coins and bills are selected from the wallet. The pupil should frequently be offered the opportunity to count the contents of her teacher’s pocket-book – that is, real money! Counting change should initially be practiced using only whole Euro amounts, and later using a combination of Euros and cents, these separated by the comma. Offered the opportunity to count the contents of her teacher’s pocket-book – that is, real money!

Change, please!

An initial understanding of invariance is prerequisite for making change. Does the five-Euro bill have the same value as five one-Euro coins, or three one-Euro coins plus one two-Euro coin? Until the pupil can fully digest the constancy of quantity through her various individual experiences, making change will remain one of her greatest puzzlements. That neither the number of coins nor their weight offers reliable information about the value of the whole can be made clear through the use of exchangeable pouches. The first pouch contains a five-Euro bill, the second several coins which total €5. Now the two pouches are compared to one another: Which is the heavier, which is the larger? And yet: With both pouches I can make the same exact purchase.

70 Yes we can!
Order, please!

A jumble of coins and bills is sorted using a wallet with numerous compartments. Somewhat more difficult is to organize these in the sequence “of few to many”. Eyes closed and a coin or bill is removed: But which was it? It is even trickier when two items of the sequence are exchanged with one another.

Yogurt-cup memory

First eat, then play! Coins are hidden under several identical yogurt cups. All coins are present in pairs, but only a single coin is hidden under each cup. Let’s play memory. The memory game is more difficult if the cups hide amounts that can be paired: E.g. do the two 50-cent pieces equal a €1 piece. Now the rule is to calculate exactly!

The Clock

with being able to interpret a clock. Much more decisive is a fundamental understanding not only of the fact that time passes continuously, but also, for instance, of how much time is consumed by each daily activity. The measurement of time, which is built on a unit of

60 minutes, defies the orderly decimal system. Pupils who have already learned the multiplication table of 5 should expand this to “12 times 5”, to enable them to grasp the 5-minute divisions of a clock, that together make up a full hour consisting of 60 minutes.

The sand trickles slowly…

As introduction to the intensive engagement with the topic of time it is ideal to make use of hour-glasses of various duration. Many pupils are already familiar with the well-known 3-minute hour-glass next to the toothbrush. However, hour-glasses of longer duration (e.g. for 5, 10 or 20 mins.) can also be used for numerous other activities, e.g. to establish the time allowed for dressing or for watching television. Hour-glasses help the pupil to gain an intuitive feeling for varying lengths of time. How long is a second? How long is a minute? How long is an entire hour?
The round form of a clock with a face can be compared to a small round pie. At the beginning of the learning process, the full hours are introduced; in the next phase, the half-hours, and ultimately the quarter-hours. The pupil cuts a pie (preferably one she has baked herself) into two halves, then into four quarters. These are rejoined and divided up again and again. Now the pupil cuts a previously prepared paper clock (without hands) into two half-hours and four quarter-hours. The teacher then explains to the pupil the relationship between the pie and the clock, perhaps with the following words: “These two halves of the pie are like the two half-hours of the clock. These four quarters of the pie are like the four quarter-hours of the clock.” After this comparison the torture for pupil (and teacher) ends – now they can nibble away: A quarter "hour" (a quarter of the pie) or even a half “hour” (half of the pie)?

Take an old clock with a simple face indicating only the hours. Remove the battery and the plastic protective cover. The use of a “real” clock has the advantage that the manual manipulation of the hour hand is accompanied by the movement of the minute hand. The clock should display the numbers 1 to 12 for the hours. The minutes should be represented by dots or dashes. Matching the actual size of the clock face, four quarter-hours and two half-hours should be cut out of paper and labelled with the appropriate fractions.

The slow hand
If the hour hand and minute hand are the same colour, paint the hour hand red. The hour hand should be shorter than the minute hand. The pupil learns to tell the difference between the “slow” hour hand and the “fast” minute hand. The entire hour should be worked through using the hour hand. If in the process the pupil is overly confused by the minute hand, this should be camouflaged with a narrow strip of paper.
The fast hand

For quarter-hours and half-hours, the paper cut-outs should be placed in the clock and discussed. The fast minute hand will now be manipulated to correspond to the paper cut-outs representing the positions for 15 minutes, 30 minutes and 45 minutes.

How much time is required for what?

For learning to understand the clock a camera is an indispensible aid. The pupil is photographed as she carries out her various daily activities: While learning, while drinking, while eating, during sports, while watching television etc. What lasts but a minute? What lasts a quarter of an hour, what lasts a half-hour, what lasts three-quarters of an hour, what lasts an hour? The photos are placed next to the corresponding paper cut-outs. Afterwards the minutes are counted using the dots or dashes of the clock as example. How many minutes are there in a quarter of an hour, how many in a half-hour, how many in three-quarters of an hour and how many in an entire hour? The paper cut-outs are labelled with the corresponding number of minutes.

From morning to evening

In order to create a structure for the day, the photos are arranged according to different time frames. The teacher holds all of the photos in her hand as if they were playing cards, the pupil draws a photo and places it next to previously prepared slips of paper labelled “morning–pre-noon–noon–afternoon–evening–night”. If the pupil cannot recognize the written words, the slips of paper can display individual symbols for the various time frames (e.g. a breakfast cup for the morning). What happens in the morning, where am I before noon, when is it midday, what am I doing in the afternoon, what time does the clock show in the evening and when exactly is nightfall?
Success, I’ve arrived on time!

Many teachers – and in this case we mean parents above all – know how nerve-racking it can be to try to arrive on time with a child with Down Syndrome. The stress is particularly great in the mornings when one has to catch a bus to school or to work.

In such situations, experience gained from the game “How much time is required for what?” can lead to a sense of general relief. In more peaceful moments (perhaps on the weekend), parent or teacher and child can jointly ponder how much time is required for which activity from the moment of rising to the moment of stepping onto the bus. So-called “time packages” can be worked out.

An example:
“Bathing or showering and brushing the teeth: 15 minutes”.
“Breakfast: 30 minutes”.
“Dressing: 15 minutes”.
“Walking to the bus: 15 minutes”.

Now, depending on when the pupil rises, four clocks are drawn by hand:

- The first clock shows when she has finished cleaning up and brushing her teeth. This is hung in the bathroom.
- The second clock shows when she leaves the breakfast table. This is placed on the table.
- The third clock shows when she has finished dressing. This is hung next to the closet.
- The fourth clock shows when she has arrived at the bus. This is placed in her satchel.

Initially the pupil will require the assistance of the teacher in order to learn to compare the hand-drawn clocks with actual time. But practice makes perfect. At the very latest, now would be the moment for the pupil to have her own wristwatch, digital or analog being a matter of taste. However, it is important that the watch be kept simple, without ornamentation, reduced to the most essential elements. In the long run, having “time packages” in the head can lead to efficient time management. At least it is worth a try (advises the author with a wink, herself the mother of a 16-year-old young lady with Down Syndrome).

The digital clock

The television guide shows digital time. For many a pupil a favourite program will be the perfect incentive to occupy herself with the so-called “digital clock”. On the very latest, on the analog clock the “afternoon” hours will now be entered alongside the “morning” hours: i.e. 13 next to 1, 14 next to 2 etc. At a glance the pupil sees which hours correspond to which. Now the television guide will be used to select a program that begins on the hour. When is 18:00? The pupil searches for 18 along the border, moves the hour hand to 6 on the clock face (the minute hand moves along accordingly) and sees the relationship between 18:00 and 6:00.
5-minute disc

Now the time has come to busy oneself with the many dots or dashes on the face of the clock. For this purpose a disc is cut out of paper and placed inside the clock, but without covering the hours displayed on the face of the clock. In accordance with the clock face, the 5-minute divisions will be entered as dots or dashes on the paper disc. It is greatly advantageous if the pupil has already learned the multiplication table of 5 (through 60). Now the dots or dashes are labelled: “5-10-15-20…to 60”. Now the minute hand comes into play: It is set to correspond with the time indicated in the television guide. But beware, a trap! As if by magic at 12 o’clock one hand has hidden itself behind the other.

Subsequent steps in the learning process:

- Programs that begin at the half-hour, e.g. 16:30.
- Programs that begin a quarter after the hour (14:15) or a quarter before the hour (19:45).
- Arbitrary broadcast times

Pupils who are not interested in television programs might perhaps delight in the schedules of busses or trains?

Like a star!

For an entire day our pupil is now followed about with a camera, just like a real star. A snapshot is made of every important, repeated activity – ranging from her rising to going back to bed, a total of approximately 10 photos. The photos are arranged in chronological order and in the process the time-frame for each activity is discussed.

- The photos are assigned to previously prepared slips of paper that show the corresponding digital times (“digital clock”).
- The photos are laid alongside the corresponding times on the clock face (“analog clock”).

Analog clock and digital clock

Analog clocks, those with face and hands, are frequently seen in wristwatches, in the kitchen or at train stations. Clocks displaying the exact hour and minute in printed form are found in automobiles, cell phones or stop watches. They are referred to as “digital clocks”. In everyday life the pupil is consciously on the lookout for analog and digital clocks. The game of clock hunting grows more interesting if the pupil, upon finding the clocks, is allowed to photograph them and place the photos in a small album.
Bingo requires a brief preparation period. Each player receives a work sheet, each with a total of six different digital printouts.

**Depending on the learning level of the pupils, these are:**
- Division into exact hour units (e.g. 14:00)
- Division into half-hour units (e.g. 17:30)
- Division into quarter-hour units (e.g. 19:15, 13:30, 21:45)
- Division into 5-minute units (e.g. 18:05, 16:40)
- Any given digital time from 00:00 to 24:00 (e.g. 20:36)

Using an analog clock with face and hands, the teacher sets a time. The pupils compare this to the digital printouts on their work sheets. If one of these printouts matches, it is crossed out. The teacher then sets the next analog time, the pupils compare this to their work sheets. The winner is the pupil who first crosses out all six digital printouts on the work sheet.

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**Calendar**

We experience a broader concept of time in daily, weekly, monthly and yearly rhythms. Pupils can often draw upon a variety of senses in experiencing the four seasonal cycles. Which fruits and vegetables do we find at the farmers’ market in the different seasons? How do flowers smell in the spring? What clothing do we need in the winter to protect us from the cold? What are the colours of an autumn tree? Which birds chirp the loudest in the summer?

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**Photos, photos**

The pupil brings along old photos from her childhood as well as current snapshots that show her during various seasons of the year, at celebrations, birthdays, on vacation. What aspects of the photos tell us when they were taken? We can make a collage of the photos based on such criteria as “older and newer” or “spring/summer” or “fall/winter”.

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Yes we can!
This is my month!

The self-made yearly calendar requires a bit of time for preparation, 32 thumbtacks, approximately 50 small slips of paper and one large piece of coloured pasteboard. The days of the week are entered at the top of the pasteboard. 31 slips of paper serve as the days of the month. They are labelled with the numbers 1 to 31. On the remaining slips of paper the important celebrations of the year, recurring and unusual leisure activities, vacations, as well as everyday activities (shopping, working, going to school) are written or drawn. At the beginning of each month the slips of paper representing the days and the respective planned activities are attached to the pasteboard with thumbtacks. At the end of each individual day the corresponding slip of paper is removed (and set aside for the following month). Through the recurring structure, the pupil will slowly familiarize herself with the 7 days of the week and the 4 to 5 weeks of the month. Parallel to this, the 4 seasons and the 12 months of the year can be pictorially represented in the form of a disc.

Weekdays and the names of the months

It is not easy to maintain an overview of the days of the weeks and the names of the months. In the first place, the names need to become familiar; thereafter, their correct sequence must be noted. If the pupil is capable of holistic word recognition, the following memory game can be an entertaining aid: Each week day is written on a small card. The cards are then laid out in the correct order and discussed. What do we do on a given day?

And now once again: “Eyes closed!”

- The teacher turns over the present day of the week. Now which is it?
- The teacher exchanges two week days. What is the correct sequence?
- The teacher turns over the previous and following week days. Which was yesterday and which comes tomorrow?
- As soon as the pupil has learned the week days, the game can be repeated with the names of the months.
Today, yesterday, tomorrow!

The correct use of the words “today, yesterday, tomorrow” is often quite confusing. The pupil can be assisted if the words are accompanied by gestures. For “today” the index finger points to the floor. For “yesterday” the index finger points backwards over the shoulder. For “tomorrow” the index finger points forward into the air.

Whenever the teacher uses the words “today, yesterday, tomorrow” with the pupil, she should simultaneously accompany these with the corresponding gestures. The pupil will therefore also be encouraged to use these gestures. They can help her not only to sort out her thoughts, but also to organize events better chronologically.

7 days, 12 months, 4 seasons – rather complicated! However, everything is much simpler when accompanied by motion. The teacher claps her hands 12 times. What does this mean? Correct, the 12 months! Then she raps 7 times on the table. Yes, this represents the 7 days of the week. And when she stomps 4 times? Then this means the 4 seasons.

Now the game is turned around and the pupil thinks up various motions (times 7, times 12, times 4). The teacher must count exactly and make a guess. Please pay close attention! The self-made calendar and the disc displaying the seasons can assist the pupil in maintaining an overview.

Measuring and weighing

Weighing and measuring, just like managing time and money, are a “mathematical problem of everyday life”. Comparing amounts and lengths, shifting from one entity to the other, is frequently very puzzling to persons with either 46 or 47 chromosomes. Prerequisite for success is, on the one hand, a fundamental understanding of invariance, and on the other hand, the decimal system.
Everything at a glance

In order to simplify the conversion of cm into dkm and m, and of gm into dkg and kg, we have worked together with individuals with Down Syndrome to develop a chart that shows at a glance how quantities in a cookbook or in an instruction manual can be converted from one unit of measurement to another. In need of a simple mnemonic aid? “Kilo” is the Greek word for thousand. 1 kilometre thus corresponds to 1000 metres and 1 kilogram to 1000 grams. Logical, yes?

Measuring

“Long, longer, the longest, short, shorter, the shortest”: Before pupils are introduced to measuring instruments, it is essential that they first understand these concepts and are able to arrange them in the correct order. In addition to length, it is also exciting from day to day to measure height. The pupil must therefore also grow familiar with the concepts “high, higher, the highest” or “tall, taller, the tallest” and “low, lower the lowest”.

Body measurements

Pupils can familiarize themselves with units of length either by measuring common, everyday objects or by measuring their own bodies. For understanding the concepts “cm, dkm, m”, measuring individual parts of their bodies can be of great assistance.

An example:

- How long is 1 cm? As long as my fingernail.
- How long is 1 dkm? As long as my hand.
- How long is 1 m? The distance from the top of my head to my knee.

Independent of the pupils’ heights, individual parts of their bodies can be measured and the sizes used for comparisons in everyday situations. “Is that book longer than 1 dkm, that is, larger than my hand?” “How large might that chest be, how long the spaghetti noodle?”
Approximation exercises

How many “hands” (i.e. dkm) do I need to measure from here to the door? Does one unit of “head to knee” suffice to measure the size of the table? Approximation exercises require a lot of movement, and this is accompanied by quite a bit of laughter, since occasionally ridiculous contortions are necessary to make the final comparison between the estimated and the actual size.

Measuring instruments

A long paper strip is all that is necessary to create one’s own metre stick. For this, the hand is laid on the paper 10 times. A tens-stick, like that for calculating, can also be used to measure distances of 10 cm. The “Measuring-Pupil” will feel like a pro as soon as he begins to work with a ruler, a metre stick and a tape measurer. It would be best in the beginning to paste over the mm divisions with white labels. These can then be removed at a later time, if necessary.

• Who can toss the beanbag the furthest?
• Which cherry seed can be spit the furthest?
• How far can the model car roll? How hard must it be pushed to reach the 20 cm mark?
• Who can build the highest tower with paper cups?

Woollen yarn approximations

The pupil and teacher together cut woollen yarn into lengths of 1 cm, 5 cm, 1 dkm, 5 dkm and 1 m. These are placed on small cards that are labelled with the corresponding lengths. Now once again: Eyes closed! The teacher places one of the pieces of yarn in the pupil’s hand, and the pupil, with eyes closed, is to estimate how long it is. The empty card provides the clue to the answer.

I am this tall!

By using a measuring rod that hangs on the wall, one can determine the current height of the pupil. This will be marked with a clothes-peg and annotated with the date. After a few months, one can make a comparison. The pupil’s foot can also be outlined on a piece of paper and the shoe size and date jotted down. What has changed over the last few months?
Handyman, take note!

The teacher plans with the pupil actual everyday situations that require measuring.

**A couple of examples:**

- “This picture needs a frame. Let’s measure it.” Afterwards teacher and pupil together buy the frame.
- “This shoe chest has to fit in the corner. Let’s measure it.” The shoe chest will also be bought afterwards by teacher and pupil together.

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**Weighing**

Both kitchen and bathroom scales are indispensable to everyday life – also, and particularly, for individuals with Down Syndrome. Cooking, on the one hand, and weight control, on the other, contribute substantially to the quality of life. The weighing of quantities is a central component of each.

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**Pack your knapsack**

What is heavy, what is light? A pupil packs a knapsack so full of objects that he is no longer able to lift it without assistance. After several unsuccessful attempts to carry the knapsack from the room, he now begins to repack it, until he has lightened the load and can now carry the knapsack away on his back. This experience should be repeated with other bags and additional objects.

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**Weight comparisons**

Once again we put the principle of invariance to the test. Does the small object really weigh as much as the larger one? The simplest scale is a coat hanger, on the 2 ends of which paper cups are attached with strings. The coat hanger is suspended on a door handle or a hall stand. Filling the cups with small everyday objects produces interesting results. The coat-hanger scale is also able to provide a clear answer to the question “which is heavier, which is lighter?” It is important that the pupil first weigh and evaluate both objects in his two hands, in order to gain an intuitive feeling for the concepts “heavier and lighter.”
1 kilogram

The pupil learns the term “kilogram” (and its abbreviation kg) by looking in the kitchen (or in a grocery store) for packages that are accordingly labelled (e.g. rice, noodles, flour, sugar). These packages are consciously picked up and held in the hand. Now this is what “1 kg” feels like. Afterwards the kitchen scale will be put to use. How does it indicate 1 kg? And when shopping at the market the large vegetable scale will be used to weigh 1 kg of apples just as exactly as it weighs 2 kg of potatoes.

I weigh this much

From the kitchen scale we now progress to the bathroom scale: The pupil weighs herself and states her weight. This, however, makes sense only if the pupil can already calculate within the range of her weight. Comparisons with other persons must be conducted with the utmost sensitivity, in order not to hurt the feelings of one who is perhaps overweight. It can be quite amusing, however, to weigh oneself, to write down that weight, and then pick up various light/heavy objects. How does her weight change when a pupil holds a heavy bag in her hand? How does her weight change when she holds a litre of milk in her hand?

dkg – who weighs how much?

If the pupil is already capable of calculating up to 100, she can be introduced to the term “decagram” (and its abbreviation dkg). Teacher and pupil together look for objects that the pupil estimates to be “light”, that therefore weigh less than 1 kg, and together they weigh the objects and write down their weights on small cards. Now the cards are shuffled and then once more assigned to the appropriate object.

Baking cakes

Weighing quantities when baking and cooking should take into consideration the numerical skills of the pupil. Only if the pupil is capable of calculating up to 1000 is it meaningful to introduce weights such as 250g. Then one can use the term “gram” (abbreviated gm). Until that time there are delicious tumbler cakes that require no scale and yet offer a first understanding of proportions between quantities.
In order for individuals with Down Syndrome to learn to estimate not only the energy contained in foods but also the amount of movement required to burn off that energy, it is simplest for them to work with a system of points. 1 point in this system represents approximately 50 kilocalories. The system also requires that food be weighed. Several examples with approximate quantities (but please, each pupil should create her own list with her favourite foods and beverages): If we assume an energy requirement of ca. 2000 kilocalories per day, that would be 40 points per day. But be careful: It is not the meals alone that must be counted, but also their preparation (above all, the oil used for cooking or in the salad), all beverages (sodas and iced teas contain up to 9 cubes of sugar per glass) and all snacks in between.

Roughly estimated, we need on the average 1 point as fuel for approximately 15 minutes of motion. (This is, of course, dependent upon the type of motion and the speed with which it is executed.) With the help of her individual list of points, the pupil can calculate her own daily caloric intake and compare this to the energy she burns through motion.

<table>
<thead>
<tr>
<th>Points</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200 gm leaf lettuce, or 200 gm strawberries, or 1 teaspoon butter</td>
</tr>
<tr>
<td>2</td>
<td>1 large apple, or 1 small banana, or 2 pieces of potatoes, or 1 wheat bun, or 2 teaspoons of marmalade, or 1 scoop of ice cream, or 1 glass of soda</td>
</tr>
<tr>
<td>3</td>
<td>40 gm noodles, or 40 gm rice, or ½ bread dumpling, or 55 gm French fries, or 150 gm potato salad, or 1 potato dumpling</td>
</tr>
<tr>
<td>4</td>
<td>200 gm chicken breast, or 140 gm salmon, or 400 gm trout, or 1 piece of fruit tart, or 10 butter cookies, or 1 small croissant</td>
</tr>
<tr>
<td>5</td>
<td>150 gm pork cutlet, or 250 gm veal cutlet</td>
</tr>
<tr>
<td>6</td>
<td>80 gm cheese with 45% fat i.d.m., or 100 gm sausage, or 70 gm salted crackers, or 1 hamburger</td>
</tr>
<tr>
<td>20</td>
<td>1 pizza</td>
</tr>
</tbody>
</table>
Marion is a 17-year-old young lady with Down Syndrome. As part of her daily life she proudly makes use of technical aids for communication, for learning-support, and for organizing her leisure time. The pocket calculator, the adding machine, the cell phone and the PC help individuals with Down Syndrome to conquer everyday situations. Through their use, these individuals generally acquire self-confidence and a feeling of inherent competence. However, the offer to introduce technical aids should not be made too soon. For every one of those individuals with Down Syndrome who has mastered basic calculations up to 100 using his own two hands, reliance on the materials offered by his own body should always take precedence over the use of pocket calculators, in order to minimize dependency. Nevertheless, for certain situations – such as for calculations beyond 100, to control change returned after a transaction, and for all of those individuals with Down Syndrome who cannot master counting and calculating skills by relying on their bodies – the use of the pocket calculator should be specifically trained.

Better separated!

With every technical aid one should pay special attention to the fact that the areas with the numbers and the functional signs are large and distinctly separated from one another. All unnecessary elements should be covered with small labels of a single colour.

Stop–Check!

When using technical aids it is crucial to place less stress on speed in order to be able to pay more attention to accuracy! This is clearly demonstrated with the pocket calculator as example: Enter a number, then stop briefly to check it against the original. Only after this step is the functional sign entered, again stop to check, then the next number, and so on, until the calculation is fully entered.
Yes we can!
Children must have particular capabilities and abilities in order to understand mathematics, achieve some outcome and have success. Among children with DS we can see big differences in mathematical skills, even if they are very small. The problem is that children without abstract mathematical thinking and imagination are generally not interested in counting. They refuse counting games, have difficulties with sustaining attention and therefore subconsciously refuse any mathematical work at all. All children must get to know the meaning of what we request and what sense it will make for them in every day’s life. Regarding mental disability the image of future or other use of mathematics is impossible. They confine in respect to time to: here and now.

Thus, the most important input for the development of teaching mathematics and development of a child’s mathematical ability is the child’s motivation to cooperate not only with the teacher, but also during so called social learning with parents or other caring people. How? I cannot answer this question myself. Motivation will vary depending on the direction of each child’s interest, therefore it is impossible to create a manual. It doesn’t work without close cooperation with the family and all caring people. It is primarily up to the parents to show their child how computation is needed in everyday life.

Example: Most children like purchase. We go to a supermarket and if we are just about to teach number 3, we buy 3 pieces of everything. We let the child put the products into the basket by himself and count the given number of pieces. At home the child helps taking out the products and separate them. The aim is creating an image of the number.

Especially children with Down syndrome tend to fail primarily because of the inability to fully concentrate on work, especially if it is not in the centre of its interest. They quickly tend to forget the mastered subject matter or they have difficulties in applying the information in another situation.

Development of cognitive abilities of children using the method of instrumental enrichment from Reuven Feuerstein.

But the instrumental enrichment program including worksheets is not available in free distribution and its use is bounded to special training. Only trained teachers can work with this program, instruments are under copyright and free copying is not allowed. Therefore, it is impossible to incorporate elements from FM into methodology without the consent of the author of the method, R. Feuerstein. There are around 80 training centers around the world, which organize courses for teachers, psychologists and parents. One of them is in Prague. The method is based on 21 training workbooks, which are called instruments. Each of these instruments deepens the child’s ability to absorb the subject matter, separately considered in context – it develops cognitive functions. It contains tasks using paper and pencils, which are aimed at other partial thought operations. Work, however, doesn’t lie in „filling in“ pre-printed sheets, but the matter is in dialogue. The teacher talks with the pupil, he brings a variety of topics and situations. He mediates the thought process that helps to solve the problem, uses „bridging“ to other situations based on experience and interest of the pupil. Then both will search for a solution. Basically each pupil’s response related to the topic is correct, but ... the teacher leads the pupil to the most appropriate answer. The emphasis is on developing speech because speech is the instrument of each intellectual activity. The child should be able to say each thinking process and each strategy it uses.

What is important is internal motivation.
Most of all the child must have a motivation to learn and a knowledge about the reason of learning this specific subject. Orientation in space is one of the basic pre-condition for success in mathematics. A defect of special perception is relatively common by children with mental deficiency and it affects the overall perception of reality. The children then have difficulties in creating relationships between objects or digits. A similar problem we find in the perception of space - objects are standing alone, without interaction. The child shows: this, that ... and doesn’t use words such as above, under, left, right, in front, behind. The imagination is limited by it too, so children often work this way: attempt and error, they cannot create relationships between objects and events and understand them as separate elements. Within this area they usually have poor vocabulary, use just pointing pronouns. First the pupils should get knowledge about the existence of space and practically practice movement in it. Their view of space is egocentric. We are working with pictures. It is mainly about narration and a description of a picture, where we notice right spatial relationships. The child learns to work with the words in front, right in front, right behind, under, above, inside, outside. Besides working with pictures and images I also create situations directly in space, where the mother, the child and myself take place. Another level is the drawing of elements into the picture according to verbal instructions. For children it is attractive to examine the functions and composition of individual devices, they like playing with building blocks, compose puzzle. A child with defect of analytical perception does not show this interest. In Czech language it is not able to join sounds into words and vice versa. Problems with mathematics can be assumed, where the analysis or dividing is essential. Work with a brick-box and games teach kids to see individual parts of a whole, to divide a whole and connect it again, to create a structure. Analysis and synthesis are the basic operations. For the child even adaptation and orientation in the world lies in the ability to differentiate and integrate. The child develops cognitive strategies leading to creating its own position or its change in approach to reality. Division of a whole into parts, whether concrete or abstract, requires given relationships between a whole and its parts and between parts. Pupils learn to distinguish between what the object or event are from. They learn to understand individual steps of the process and that the whole can be divided in various ways, but its uniformity is maintained. When pupils are working with pictures or with words, they compare. Pupils learn to work systematically, to judge the differences and to determine their characteristics. We judge and compare according to all criteria and characteristics, from concrete objects to abstract concepts. Pupils can incorporate individual information into a whole only through mutual comparison. The aim is to teach them to compare any new information spontaneously with the already known and to maintain its stability even if the objects are different (e.g. 5 is always 5, no matter whether they are apples, cars, houses, etc.). I am using many practical exercises, for example, I put a glass of milk and a bowl with sugar in front of the child. We then are seeking what they have in common (color, both the food) and what’s different (taste, form, origin, use ...). We compare the toys, figures, devices, and use the words in front, right in front, right behind, under, above, inside, outside, and any other spatial relationship. Pupils learn to compare and generalize the criteria for comparison. They learn to sort objects according to similarity with a given model or momentary importance. They find examples similar to or different from a pattern in one or more characteristics alone.
pictures, animals ... first real objects, then according to a picture, then only according to a verbal instruction. We can also include comparing the value of numbers, quantities, weights, distances. We expect the pupils to have the ability to compare, recognize and differentiate. They learn to generalize and organize collected data into superior categories. Criteria for classification is created, they learn that a choice depends on an actual need. Analogous objects can so be repeatedly inserted into different groups according to given criteria.

**Examples of exercises:**
- Sort simple objects such as cubes, crayons, small wheel according to a criteria (color, size).
- Determine the principles or options of sorting.
- Search different elements.
- Illustrate or express the result of sorting in various ways.
- Learn more complex sorting according to more criteria.

Manipulation with different materials leads to understanding the value of number, which is an essential presumption for successful understanding of mathematics in elementary school. It is necessary to introduce the concept of numbers, so that the child can imagine the name and shape of the number and the number of elements, which it indicates. Children can learn the numerical series easily like a poem, but they have no concrete idea of the particular number, do not know the sequence, what is one more. A good task for the pupil is to create groups with a given number of elements. Initially they are very simple groups, only two, three pieces so the operation becomes automatic. The child must understand the concept of numbers and learn to manipulate with elements. Groups may be heterogeneous, the number of elements is deciding. They learn that two groups with two elements make four, three groups make six, etc. The shape and color of elements is changing but it does not affect the number! The basis for understanding mathematics is to understand the decomposition of number. The child should be able to add any number to ten (complete the sum of every number in series 1-10 to ten). This ability can then be transferred to any relationship. In the Czech Republic we have very good experiences with teaching mathematics to children with special needs (DS included) according to the method Step by step (author and trainer is Netty Engels, The Netherlands). This methodology is based upon the routine dividing of numbers 1-10. When the child is certainly doing this (with numbers 1-10), he can do the same with the next numbers (1-100, 1-1000...). Using the method of instrumental enrichment from Reuven Feuerstein, the child never uses his fingers! He can use some materials for better understanding of the value of numbers, like a special abacus (counter tool). But later (after reaching some level of competences) the child is not allowed to use any tools or materials, he has to count by heart/ by memory. This is called cognitive mathematics and it is based upon principles of the Reuven Feuerstein method and long-time practical experiences of Ms. Engels with children with DS. Very important is good cooperation of the teacher with pupils for effective and successful results in teaching children with DS. It always depends on the teacher, how close he can get to his client, how much he can understand him and make him cooperate. We can talk about it only in case if the child believes that the tasks are made for him. The child knows what the aim of the work is, why it does this specific work, and what benefit it has for it. Motivation must be appropriate to the age and intellectual abilities of the child.

**References**
- MÁLKOVÁ, G. Zprostředkováno učení (Mediated Learning), Prague: Portal 2009
- LEBEER, J. (Ed), Programy pro rozvoj myšlení dětí s odběrům vývoje (Inclusive education of children with developmental difficulties through basic Skill Instruction and Developmental Education), Prague: Portal 2006, ISBN 80-7367-103-4
Montessori pedagogy is essentially inclusive pedagogy. Montessori pedagogy is inclusive pedagogy by nature of its objective

Trained as a doctor, Maria Montessori made the transition from medicine to pedagogy in the years 1896-1906. She no longer regards only the organically ill child as in need of help, rather she develops an understanding for the problems of the aberrant child, i.e. the one developing outside the mould, and recognizes the necessity of coming to the aid of these children. Now she proceeds to put into action her ideas for assisting the child, that child, to discover his real self. She therefore calls for the elimination of social abuses by encouraging educational reforms, particularly reform for the training of children who face exceptional challenges in life. She gives lectures on these topics and in 1900 assumes the directorship of a medical-pedagogical institute that includes a school serving as model for the training of teachers of handicapped children; she continues her work on the development of a specific method for observation and care (Helmut Heiland, Montessori, pp. 34 f.), its goal being “to assist them towards independence from others and towards human dignity” (quoted from Helmut Heiland, a.a.O. pp. 38).

Thus “a plus factor of the Montessori method is the general validity of its procedure: It makes no difference if the person in question has been diagnosed as mentally deficient or highly gifted. This procedure does not even require the ability to speak. It can therefore provide important insights into the polarization of attentiveness in persons with the severest of handicaps” (André Frank Zimpel, Der zählende Mensch, Göttingen 2008, p. 83).

Montessori pedagogy is inclusive pedagogy by nature of its method

The Montessori Material

Generally speaking, Maria Montessori is primarily known through the so-called Montessori Materials. She developed materials that contain, among other things, mathematical ideas that are self-explanatory. “The materials address not only the senses of sight and touch, but also those of hearing and balance. For Montessori, to learn with all of the senses does not mean to address all of the senses at once. Quite rightly she is more concerned with isolating the individual senses, for only in this way will the children become aware of them” (André Frank Zimpel, Der zählende Mensch, a.a.O. S. 83). From these materials she shapes a richly stimulating world of learning possibilities and learning impetuses — exactly coordinated to the needs of the individual child. Through its form, the material should absorb the attention, include error-checks to enable independent learning and isolate an individual property (weight, form size) in order to achieve clarity and the ability to discriminate. The single duty of the teacher is to establish contact between the child and the material, to enable and to accompany while observing, and thus to assist the child to concentrate and to work. In this case the child, not the teacher, is the active ingredient; the objects, not the teacher, the priority. Maria Montessori sees nothing but the respective child as centrepiece of her pedagogical activities, whose exclusive goals are to train the child to work with the materials of the prepared environment and to assure that a child who is absorbed in his work is disturbed by no one else.
A prepared environment is an inclusive environment

The “prepared environment” that is thus individually created for the child is comprised of all those materials that issue from his respective stage of development and from his other individual needs. This environment should have a “fixed dimension”, since interest and concentration grow in the same degree that the confusing and superfluous are eliminated. The child’s environment is therefore prepared in such a way that it is full of interesting opportunities for activity, which in turn clears the path towards work that should reveal greater things than those previously deemed sufficient for a child of this age. This environment should offer the least possible resistance with respect to learning and working, and thus it is the duty of the teacher to diminish or entirely eliminate every possible obstacle that stands in the way of the child’s progress. At the same time, this prepared environment must also offer and enable freedom. By exactly observing the child, the teacher should be able to detect the sensitive phases of the child’s development and to take advantage of these in a supportive fashion by introducing suitable learning materials.

From this description of the “prepared environment” it is clear that inclusion can be possible within such a context. Within the social environment of a learning group or a school class a personal learning sphere is also fashioned for the handicapped child in that he is allowed to develop according to his own skills, his own speed and his own needs. The Montessori pedagogy follows the principle of individualisation: No child is like the other, every life story is unique, including that of the child with Down Syndrome. There is only one adaptation here, namely the adaptation of the prepared environment to the child, not, however, of the child to the social environment of the respective learning group. The respective, individual child is the centre-piece of interest, not the group or the position of the child to or within that group. In contact with the materials and his work with them, the child thus develops according to his own internal ground plan which is to be respected and encouraged by the teacher. Working together with other children is certainly desirable and should be encouraged, however, always according to the rule that actual work must be enabled and not disturbed.

Independence and freedom in choosing the materials encourage the development of good working habits and the mental activity of the children: They learn to plan, prepare, arrange, generalize, adjust to others, consult with one another and to work together. Nevertheless, among pupils with Down Syndrome one can sometimes observe a deep inhibition to confront anything new. It is therefore very important that the teacher successfully awake the interest of the pupil for the new task and assure that he enjoys this work.

In using the Montessori Materials the rule applies: If the child still makes mistakes after one lesson in working with the material – that he should not be corrected, as he should in no case become disheartened. On the contrary, timid children are to be encouraged and con-firmed in their efforts. All of the materials include error-checks and thereby enable actual and perceived independence from the control of the teacher.

Montessori’s pedagogical ideas did not achieve the same renown as her Materials. Although her concept of working freedom was thoroughly accepted in educational institutions, her other guiding principles were not widely circulated – with the exception of: “Help me to help myself”. Exactly these pedagogical ideas should find foothold independent of the work with the Montessori Materials, for they contain important thoughts that can be very useful when working with individuals with Down Syndrome.

“Great work” in Montessori and “Flow”

In 1990 the psychologist Mihaly Csikszentmihalyi published a research project for which he applied modern psychological methods to investigate the question of when people feel the happiest. The study was carried out worldwide with persons of every age group, of the most diverse ethnicity, from the most dissimilar of cultural circles, of various social classes and independent of citizenship and sex. The surprising outcome was that all of these persons virtually unanimously agreed on what an activity must include for them to feel a sense of joy or happiness. As a result of the research, Csikszentmihalyi describes an effect that he refers to as a flow-experience and in which he distinguishes the following elements. The experience usually occurs when we feel equal to the task at hand (correlation between the goal and the skills of the individual).
1. Concentrating on the task at hand must be guaranteed (concentration).
2. The task contains clear objectives.
3. There is immediate feedback.
4. The intense, effortless dedication to executing the task banishes all the worries and frustrations of everyday life from the consciousness.
5. One has a feeling of control over the activity.
6. Concern about oneself disappears, yet paradoxically after the flow-experience the sense of self resurfaces strengthened.
7. The feeling for the passage of time is altered during the event.

When one considers, with respect to the creation of a “prepared environment”, that it is the task of the teacher to assure the correlation between the goal and the skills of the child, that the clear objectives are guaranteed by the Montessori material itself, and that the error-check inherent to this material provides immediate feedback, the ready conclusion is that both Montessori and Csikszentmihalyi are speaking of the same phenomenon. By flow-experience Csikszentmihalyi means “something like an activity delirium. As in the Montessori experiment, attentiveness, motivation and the environment combine in productive harmony” (André Frank Zimpel, Der zählende Mensch, a.a.O. p. 84).

For teaching individuals with Down Syndrome, this would mean that they, too, could experience great pleasure through learning experiences – provided one works according to the principles of the Montessori pedagogy. Experiences based on practical application of the Montessori methods have in fact shown that pleasure can be very clearly observed in individuals with Down Syndrome. Thus it can come to pass that the child with Down Syndrome, immersed in his work, will suddenly be perceived differently by others. No longer in the forefront is the chromosome alteration that impedes him in one way or another or even makes his task impossible; now one sees only his dedication to his work, his concentration, diligence and stamina.

The limitations of the person fade into the background, are no longer significant in the eyes of his classmates, and a process of “normalisation”, of inclusion begins to develop. In the social realm, respect grows for the achievement of this person, as it may be taken for granted that everyone is subject to limitations in what he does, which may gradually lead to distinctions, but which does not generally make meaningful activities impossible. Often, however, a handicap is regarded as a limitation that would seem to preclude the ability to accomplish anything meaningful. These inclusion-phenomena also transpire in the instructional realm and precisely when the child with Down Syndrome is occupied with other instructional objects or materials than the majority of the learning group.

**Sensory material and materials for mathematics**

With the so-called “sensory material” valuable opportunities for pragmatic experiences are expanded for the child, or his previously gained environmental experiences can be intensified through these. During the pre-school years these as a rule are bound to specific situations and are not dealt with by conversion into a formal language or by solving calculations, rather by the development of adaptive, flexible solution management that addresses the problem at hand. The sensory material should pave the way for actual mathematical exercises. It should lead to the isolation of the senses (e.g. the sense of sight), provide knowledge about the characteristics of objects (size, dimensions) and enable associative thinking, e.g. the building of pairs, gradations and contrasts. The children should reach the stage of being able to develop rules based on their experiences. Repeated use of and contact with the material will additionally train their motor and sensory skills. Actual work in the area of mathematics begins in the range of 0-10 with various materials that simultaneously represent the respective quantity, but which can also be immediately combined with numbers and even lead to the first simple mathematical operations. With Montessori it is first the question of exploring the number system, and then attempting calculations. Once the number system has been fully comprehended, calculating can, for instance, be understood as an abbreviated form of counting. Thereupon follows work with further materials that occupy themselves with practicing linear counting, the introduction of the decimal system by means of the golden pearl material and the operations of adding, subtracting, multiplying and dividing, initially using this material, but later in expanded form with further, more complex materials. The very great diversity of materials presents mathematical formulations as ever-new and meaningful opportunities for activity. The work also does not stop with numbers ranging up to ten or twenty, with which the usual supportive pedagogy spends quite a lot of time, rather progresses quickly to the larger numbers of one hundred and...
above, on to a thousand, so that these children initially become acquainted with information about these quantities. Nevertheless, experience shows that it is questionable whether pupils with Down Syndrome actually comprehend the mathematical background of the materials within these ranges of numbers. The danger arises that during the various lessons based on imitation of a model that only motions are practiced and behaviour trained, without an understanding of the underlying meaning. This can lead to so-called “sluggish knowledge” that cannot be transferred to other situations and put to use, or that cannot be applied in daily life. It can also happen that teachers have the false impression that the learning material has been understood. How else can one explain the frequently encountered phenomenon, particularly with respect to young adults with Down Syndrome, that after leaving school and in the course of their professional activity, large areas of the mathematical skills and knowledge they acquired in school disappear. A further possible difficulty arises with motor skills, in the handling of very small or partially round materials. Here one must focus intensely on the interplay with the material, as it lays the foundation for understanding the actual mathematical process.

Observation

Particularly important in the Montessori pedagogy is observation of the pupil. The attention of the educator is here directed above all towards the individual ways in which the pupil carries out his tasks, his stamina, his organisation of activities, furthermore, towards recognition of his individual inclinations, in order to offer him stronger support. By inquiry into the developmental phase and beyond, this type of observation attempts to “fathom the secret of the child”, whose development can be accelerated only if one is attuned to the child’s most inner self. Montessori also stresses that one must be prepared to observe phenomena that are not conspicuous.

Briefly summarized, the Montessori pedagogy may be viewed as a “cosmos” of (1) a finely honed pedagogical system and (2) an abundance of materials that are exceptionally and particularly well suited to aid and support individuals with Down Syndrome. Through this system one can successfully rise to meet Montessori’s challenge of “prepare me for life”, which means not only to recognize individual talents and to awaken diverse interests, but also to mobilize resources and make individuals with Down Syndrome able to cope, ready to achieve and socially adept.
that they can use this material to solve mathematical problems successfully. The Numicon material consists of colourful plastic shapes, pegs (that resemble large Lego forms) and various other objects (baseboards, a sack for feeling shapes, rotatable discs, number array) that can be employed for working through various activities. This should be a tool for the development of the child’s understanding of numbers. In using it, the child should develop skills and concepts that allow him to apply these to new situations that require an understanding of numbers. To achieve this, the Numicon authors stress how important it is to establish connections between the activities and real-life situations. The exercise program therefore links together the areas of sight, sound, touch and communication. This is particularly meaningful for children with learning disabilities since they cannot establish such associations and generalisations themselves. Since the Numicon designs and pictures enable children to “see” how many objects lie before them, inasmuch as they are arranged in systematic and recognizable patterns, the exact observation and examination of the patterns, likewise touching and connecting the pictures, teach the children to grasp the relationships between the numbers.

Here one takes advantage of the fact that people in general can retain vivid pictorial information better than auditory information or abstract, obscure displays. Even those children who are not capable of understanding abstract number concepts through Numicon can nevertheless learn to perform simple calculations by means of the pictorial representations in the drilled activities. For a successful transition to arithmetic assumes that children can proceed beyond simple counting and grasp numbers as entities. “Seven” must then represent an entity for the child, and not a counting process. In order to lead children away from simple counting, one must successfully demonstrate to them that they can determine how many items are in a collection without having to count them. In this case patterns play a decisive role.

With **Numicon the child quickly learns**

- To recognize forms, patterns and amounts when reencountered,
- to comprehend amounts through the activities, without having to count, and
- to develop conceptualizations.

As the children come to learn the patterns, they should be capable of seeing them before their mental eye. If they can do this and have formed a mental image of the patterns, they are in turn capable of solving mathematical problems through these images. The objective is to assure that for a specific number each child develops a conceptualization that includes the following:

**the Numicon image**

- a position on the number array,
- a number or a succession of numbers,
- a word,
- pictures where the number is represented in any desired order,
- counting experiences,
- and in connection with this incidents of daily life.

The various activities [...] now offer a number of “key words” whose usage should be practiced increasingly as accompaniment to the activities. Incorporated into normal, everyday life, they should promote and develop conceptualization.

The game potentials of the material give children the chance to explore these in their own particular way. Children should freely experiment with Numicon even before they are ready or able to use the material with a specific goal or purpose in mind, and this important game phase cannot and should not be cut short. In this way the children become familiar with the material and enjoy making use of it. Likewise, while at work on a specific activity, it is always permissible to explore the materials. Through this the child can develop and demonstrate his own surprising ideas. It is for this reason that most children who work with Numicon develop a confident and self-assured attitude and feel drawn to the material and the subject matter. In addition to the game potentials of the system, all activities are specifically provided with games that offer further opportunities, above all at home, to acquire or reinforce pre-mathematical experiences.

With respect to time and organization, both of these possibilities, so utterly valuable for achievement in learning, are difficult to manage at school or during special instruction, – and yet both are ideal for children with mathematical weaknesses. Here one is offered a training opportunity that is particularly well suited for practicing unobtrusively in the informal setting of daily life. Each activity of this system can be associated with everyday items and actions. For children the system possesses a high degree of challenge, haptic qualities that invite one to touch and play, a size that not only caters to children having problems with motion or perception, but also allows use of the materials without disruption. The material is clear, well structured, presents the number system with clarity, and visually demonstrates the connection between the numbers (addition and subtraction are easier to understand). Furthermore, the logical set-up of the materials enables the children to direct their attention to that which is essential. The use of the Numicon materials requires no special training. The directions are simple, as well as easy to understand and to follow. For this reason it is exceptionally well suited for parents who wish to work with their children, whose success is in turn guaranteed by exercises in a necessarily compact form.

All of which enables “an anxiety-free, interest-oriented encounter with old, recurring and sometimes new materials”. The system stresses taking the time to develop mathematical skills and ideas before taking the time and effort to transform these ideas into written symbols. Total attention is focussed exclusively on the material and the newly learned skills. With this material the foundation is first laid for the full scope of cognitive skills; only later are specific competencies trained and refined.
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KONTAKT

Austria
Verein „Hand in Hand”, Leoben
E-mail: institut@down-syndrom.at

Germany
Deutsches Down-Syndrom InfoCenter, Lauf
E-mail: ds.infocenter@t-online.de

Italy
Arbeitskreis Eltern Behindelter (AEB)
Associazione genitori di persone in situazione di handicap,
Bozen/Bolzano
E-mail: info@a-eb.net

Czech Republic
Společnost rodičů a přátel
dětí s Downovým syndromem, Praha
E-mail: downsyndrom@centrum.cz

Denmark
Professionshøjskolen UCC, København
E-mail: ucc@ucc.dk

Romania
Fundaţia Centru Educaţional Soros, Miercurea Ciuc
E-mail: sec@sec.ro